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EXPERIMENTAL INVESTIGATION OF TEMPERATURE FEEDBACK CONTROL SYSTEMS APPLICABLE TO TURBOJET-ENGINE CONTROL

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SUMMARY

Two basic temperature feedback control systems were investigated as means of controlling tailpipe gas temperature of a turbojet engine during transient operation in the high-speed region.

A proportional-plus-integral control in a temperature - fuel-flow control system provided satisfactory transient response to a desired step increase in temperature. For a temperature - exhaust-nozzle-area control system, it was necessary to add nonlinear components to the basic proportional-plus-integral control to provide satisfactory temperature response during transients.

Several criteria for selecting control-loop parameters for optimum transient response were investigated. For the temperature - fuel-flow control system, minimization of time integrals of either the square or absolute values of temperature error seemed to be more selective than other criteria in determining optimum control-loop parameters. For the temperature-area control system, none of the criteria proved adequate in selecting optimum loop gain, but they did indicate a choice of control integral time constant.

Engine dynamics in the high-speed region were determined by synthesizing transfer functions to match experimental frequency-response data. Over the control operating region investigated, engine temperature - fuel-flow dynamics could be satisfactorily represented by first-order terms and a dead time. Similarly, engine temperature-area dynamics could be represented by first-order terms and a dead time.

INTRODUCTION

Most control systems for turbojet engines are based on use of engine speed as the primary controlled variable. Engine temperatures have become important mainly through use as damage-prevention limits and in acceleration schedules. Emphasis on speed control is, in part, due to the ease of measuring speed and the reliability of speed measuring devices.

In order to provide satisfactory control in the high-speed operating region, it has usually been necessary to add a temperature-limiting device to the speed control system to prevent damage to turbine components. Instead of using temperature limiting as an auxiliary to a speed control system, it is possible to use temperature as the primary controlled variable. This report is concerned with the experimental investigation of two basic temperature feedback control systems, one using fuel flow to control temperature and the other using exhaust-nozzle area to control temperature. Although a temperature-area control would not be feasible as the primary engine control system, it would have application as part of a two-loop system which includes speed - fuel-flow control.

The object of this investigation was to determine some of the characteristics and capabilities of temperature feedback control systems for turbojet engines. The ability of the control system to control temperature during transient operation was evaluated by considering time integrals of functions of temperature error as well as overshoot ratios and response times.

APPARATUS

Engine

A turbojet engine with an axial-flow compressor, vaporizing-type fuel injectors, and a two-stage turbine was installed in a sea-level static test facility for this investigation.

Fuel System

The original fuel system of the engine, with the exception of the flow dividers and injectors, was supplanted by a research-facility fuel control which was designed for very rapid fuel-flow changes. This control, described in reference 1, consisted of a reducing-type differential-pressure regulator that maintained a constant pressure drop across a throttle valve. An electrohydraulic servo system was used to vary the throttle-valve position linearly with input signal. Throttle area varied linearly with throttle-valve position and thus fuel flow was proportional to input signal.

Frequency response of throttle-valve position to input signal, given in reference 1, shows that this type of fuel control would not contribute significant dynamics to the control system within the frequency range investigated.

Variable-Area Exhaust-Nozzle System

A sketch of the variable-area exhaust-nozzle system is shown in figure 1. It consisted of dual butterfly valves mounted in the tailpipe about 9 feet downstream of the turbine. The shafts of the two valves were connected by a mechanical crossarm-type linkage. The upper shaft was driven by the power stage of an electrohydraulic servo system. Area variations from 73 to 113 percent of rated area were obtainable with this system. This exhaust-nozzle system was designed for fast response without consideration of good thrust characteristics.

Figure 2 shows the frequency response of the lower butterfly-valve position to a sinusoidal input signal for a total area excursion of about 10 percent of rated area.

Instrumentation

Engine speed. - To measure engine speed, a magnetic pickup was installed in the compressor housing opposite a row of steel compressor rotor blades. Each blade passing the pickup produced a voltage pulse which was fed to an electronic circuit whose output was a voltage proportional to the number of pulses per unit time. This voltage provided a good indication of instantaneous engine speed during transients, since the pickup and electronic circuit had no measurable dynamics in the frequency range of interest.

For steady-state speed measurement, the pulses from the magnetic pickup were fed to a digital counter which determined and displayed actual engine rpm when the proper time interval was selected for the counting cycle.

Tailpipe temperature. - Four rakes, each consisting of four chromel-alumel thermocouples, were used to measure tailpipe gas temperature during transients. Adjacent thermocouples on each rake were connected in series to form eight pairs. From one to eight pairs were paralleled to provide the input signal to the temperature preamplifier.

As a compromise between fast response and durability, 22-gauge wire was used for these thermocouples. With this size wire, the time constant of the thermocouples was approximately 0.3 second in the region of operation selected for the control study.

Thermocouples on two additional rakes, similar to the ones just described, were connected to a self-balancing potentiometer for steady-state temperature measurement.

Fuel flow. - The position of the throttle valve in the fuel control was measured by means of a linear variable differential transformer. The resulting position signal was the best obtainable indication of fuel flow during transients. Actual fuel flow into the engine is affected by the dynamics of the fuel manifold and flow dividers between the fuel valve and the engine.

A rotameter was used for steady-state fuel-flow measurement.

Exhaust-nozzle area. - A rotary variable differential transformer was used to measure the position of the lower butterfly valve. The signal obtained was used as an indication of exhaust-nozzle area during transients, although actual area did not vary exactly linearly with position of the butterfly valves. 4212

For steady-state area measurement the position of the lower butterfly valve was indicated by a precision direct-current meter which measured the voltage on the arm of a linear wire-wound potentiometer connected to the lower shaft. Accurate area determination could be made by using the meter reading and an area calibration curve.

Recording equipment. - Transient data were recorded on a direct-writing six-channel oscillograph recorder. The recorder pens, when used with equalizing amplifiers, had a frequency response essentially flat to 100 cycles per second. A recorder chart speed of 25 millimeters per second was used.

For sinusoidal data a galvanometric light-beam oscillograph recorder was used. Flat frequency response beyond 100 cycles per second, high sensitivities, and high chart speeds were obtainable with this equipment.

Analog computing equipment. - Circuits for calculating time integrals of the square and absolute value of temperature error were set up using linear and nonlinear elements of an electronic analog computer. These circuits are shown in figure 3. A two-stage R-C filter was used to reduce noise in the temperature-error signal. An electronic multiplier performed the squaring operation. The integral values obtained were used in evaluating transient response of controlled temperature, as will be discussed in the section Controlled Engine Data.

CONTROL SYSTEMS

A block diagram representative of the basic temperature feedback control systems used for this investigation is shown in figure 4. In these systems the indicated temperature signal was compared with the desired temperature reference level, and the difference or error signal was applied to the input of the control. The control modified the error signal according to the control transfer function. The resulting control signal was applied to the input of the fuel or area servo system, which changed fuel flow or area until the temperature-error signal became zero. Provision for step change in temperature reference level was included for transient operation.

Temperature - Fuel-Flow Control

The temperature summing network and control components for the temperature - fuel-flow control system were set-up on an electronic analog computer as shown in the circuit diagram of figure 5(a). Control gain and integral time constant could be varied separately. Also shown in figure 5(a) is a rate-limiting circuit that was inserted at point B in the control loop to limit the rate of change of fuel flow. The effects of slower fuel-valve action on control-loop-parameter selection will be discussed in the section "Rate-limited fuel control."

Temperature-Area Control

Figure 5(b) shows the computer circuit diagram of the temperature summing network and control components used for the temperature-area control system. The control circuitry for this system was more complex than that of the temperature - fuel-flow control system. Because of results encountered in a previous speed-area control-system study (ref. 2), it was found necessary to add nonlinear components to the control. These components were a diode limiter on the input to the area servo control, and a relay and relay triggering circuit.

During the previous speed-area control-system study (ref. 2), it was found that, when the area butterfly valves opened to a position corresponding to approximately 113 percent of rated area, further increase in input signal to the area servo system did not increase area. This was due to the limitation on maximum area obtainable with the butterfly valves fully open. Thus, when operating with a closed-loop system and a proportional-plus-integral control, the control output signal at times would exceed the area servo-system input level corresponding to the saturation limit (113 percent rated area). When this happened, an excessive charge would build up on the integral component of the control, which delayed the closing of the area valve when the error signal decreased. This produced poor control performance, that is, large overshoots.

For the temperature-area control-system study, the bias voltages of the limiter diodes were adjusted to limit the input signal to the area servo system to values corresponding to approximately 85 and 113 percent of rated area. The maximum area was determined by the particular area-valve design and tailpipe that were used. The minimum area was arbitrarily chosen.

The bias voltage of the relay triggering circuit was adjusted so that the relay closed when the input signal to the area servomotor control decreased below the value corresponding to 85 percent of rated area. Closing the relay, in effect, stopped the integral action of the control by adding an equal but opposite-sign error signal to the input of the integrator. A similar triggering circuit could have been added to operate the relay at the maximum area limit but was not deemed necessary for this study because the initial and maximum excursion of the area valves was toward the closed position.

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PROCEDURE

The experimental program consisted of (1) a determination of engine dynamics in the high-speed region of operation and (2) a study of controlled engine transient response and control stability limit for the two basic temperature feedback control systems.

Engine dynamics were obtained from frequency-response data. Sinusoidal input signals were applied to the fuel control and area control, and responses of engine variables were recorded. Frequency-response plots were made and analytical transfer functions, made up of engine and instrumentation dynamics, were fitted to these plots.

The engine was operated on control using the systems shown in figures 4 and 5. Step inputs were applied to increase the temperature reference level an amount equal to approximately 7 percent of rated temperature. Control gain and integral time constant were varied over a range of values, and responses of engine variables were recorded. Time integrals of the square and absolute value of temperature error were calculated and recorded. A rate-limiting circuit was added to the temperature - fuel-flow control system, and the same procedure followed. The stability limit for both control systems was investigated by increasing control gain until the system became and remained oscillatory.

RESULTS AND DISCUSSION

Engine Dynamics

Response of temperature to fuel flow. - Experimental data representing the frequency response of indicated tailpipe gas temperature to indicated fuel flow at constant area are shown in figure 6. These data were taken in the same region of engine operation as selected for the control study. The data shown in figure 6(a) and all subsequent amplitude-ratio data have been normalized (that is, the product of all constant gain terms have been set equal to unity).

The normalized temperature - fuel-flow transfer function corresponding to the data of figure 6 can be represented by

$$\left.\frac{\Delta T_i}{\Delta w_f}(s)\right|_A = \frac{(1 + a\tau_e s)(1 + \tau_1 s)G(s)e^{-t_{d,f}s}}{(1 + \tau_e s)(1 + \tau_2 s)(1 + \tau_t s)} \tag{1}$$

where $\frac{1}{1 + \tau_t s}$ represents the thermocouple dynamics (symbols are defined in appendix A), $G(s)e^{-t_{d,f}s}$ represents the fuel-distribution-system and combustion dynamics, and the remaining terms are included in the actual temperature - fuel-flow engine dynamics. Combustion dynamics, usually considered part of engine dynamics, could not easily be separated from fuel-distribution-system dynamics because of the difficulty of measuring actual fuel flow into the engine during transients. The terms $\frac{1 + \tau_1 s}{1 + \tau_2 s}$ were included to represent a long time-constant phenomenon believed due to a heating-expansion effect in the turbine section. This phenomenon has usually been neglected in previous dynamic studies.

In figure 6 are shown calculated frequency-response curves of equation (1). Numerical values for the various terms were obtained by graphical methods described in appendix B.

Response of temperature to area. - Figure 7 shows data representing the frequency response of indicated tailpipe gas temperature to exhaust-nozzle area at constant fuel flow. These experimental data were also taken in the same region of engine operation as selected for the control study.

The normalized temperature-area transfer function corresponding to the data of figure 7 can be represented by

$$\left.\frac{\Delta T_i}{\Delta A}(s)\right|_{w_f} = \frac{(1 + b\tau_e s)(1 + \tau_1 s)\ e^{-t_{d,a}s}}{(1 + \tau_e s)(1 + \tau_2 s)(1 + \tau_t s)} \tag{2}$$

where $\frac{1}{1 + \tau_t s}$ represents the thermocouple dynamics and the remaining terms represent the actual temperature-area engine dynamics. Also included in this transfer function are the two long time-constant terms previously mentioned.

Calculated frequency-response curves of equation (2) are also shown in figure 7. Numerical values used for the various terms were also obtained by graphical methods described in appendix B.

Stability-limit check. - From the frequency-response plots, predictions of stability limits for the basic temperature feedback control systems can be made. For the temperature - fuel-flow control system, the data presented in figure 6(b) indicate that a 170° to 180° phase shift occurs in the frequency range of 2.0 to 2.2 cycles per second. Considering negligible phase shift in the fuel system and less than 10° phase shift in the control (for $\tau_c \geq 0.5$ sec), a total open-loop phase shift of 180° would occur in this same frequency range, 2.0 to 2.2 cycles per second. From the amplitude-ratio data presented in figure 6(a), the loop gain at instability should be in the range of 5.7 to 6.9. For this report, loop gain is defined as the product of all the frequency invariant gains around the loop.

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Recordings of engine speed and tailpipe gas temperature taken during experimental determination of stability limit for a temperature - fuel-flow control are shown in figure 8. The loop gain necessary for sustained oscillation was 6.3 and the oscillation frequency, determined from the temperature trace, was approximately 2.2 cycles per second. Thus the stability-limit predictions from open-loop frequency-response data are substantiated very well by experimental measurements.

Similar predictions can be made from the temperature-area frequency-response data; however, the area-system dynamics must also be considered. For this control system, loop gain at instability should be 40 and oscillation frequency should be 7 to 10 cycles per second. Experimental determination of stability limit for this system was unobtainable due to noise limitations of the temperature signal at the high loop gains required.

Controlled Engine Data

Temperature - fuel-flow control. - Good control performance during transients depends on proper selection of control-loop parameters (that is, in this case, loop gain and control integral time constant). Two often-used criteria for evaluating transient response are overshoot ratio and time-to first zero. Figure 9 shows plots of these criteria (defined on fig. 10(a)) obtained from transient-response recordings when operating with a temperature - fuel-flow control. It is often difficult to select the best combination of control-loop parameters using these criteria. This is borne out by examination of the transient-response recordings presented in figure 10.

Figure 10 shows transient responses of fuel-valve position, temperature error, and engine speed to step changes in desired temperature. The transient of figure 10(a) was obtained using a control integral time constant τ_c of 0.25 second and a loop gain K_L of 1.14. This combination of control-loop parameters gave a temperature overshoot ratio of 1.12 and a time to first zero of 0.46 second. The time to first zero could be reduced by increasing the loop gain; however, this was accomplished only at the expense of greater overshoot ratio, as shown in figure 10(b). For this transient, τ_c was kept at 0.25 second and K_L was increased to 3.03. The time to first zero was reduced to 0.23 second, but the overshoot ratio was increased to 1.58. This overshoot is considered too great and the response too oscillatory for good control performance.

Figure 9 indicates that at a constant loop gain (greater than 2.5) the overshoot ratio can be decreased without greatly affecting the time to first zero by increasing τ_c. Figure 10(c) shows the transient responses for τ_c of 4.0 seconds and K_L of 3.03. The overshoot ratio was decreased to 1.10 and the time to first zero was increased slightly to 0.26 second. According to the plots of figure 9, these criteria values would indicate good control performance. However, a closer look at the temperature response in figure 10(c) shows a long drift-in or settling time. For many control systems this would be considered undesirable. Thus, this example shows a shortcoming of using the overshoot ratio and the time to first zero as criteria for selecting control-loop parameters for good control performance.

A discussion and evaluation of criteria for optimizing transient response of control systems are given in reference 3. Two of the criteria discussed in this reference were used in this temperature control-system investigation. These were the time integrals of the square and absolute value of temperature error. Circuits for automatically calculating these integrals during transients were set up on an electronic analog computer, as shown in the diagram in figure 3.

Shown in figure 11 are plots of the integral criteria values obtained during transient operation with a temperature - fuel-flow control system. The curves which give the least values for the integrals indicate the approximate values of control integral time constant and loop gain to use for optimum transient response. The criterion $\int_0^\infty T_e^2\,dt$ indicates a range of τ_c from 0.75 to 1.0 second and of K_L from 2.5 to 3.5 as giving optimum transient response. The other criterion $\int_0^\infty |T_e|\,dt$ indicates a range of τ_c from 0.5 to 0.75 second and of K_L from 2.0 to 3.0.

Although these two criteria indicate slightly different ranges of control-loop parameters for optimum transient response, both choices of optimum response are characterized by a satisfactory compromise between overshoot ratio and time to first zero.

Transient responses for τ_c of 0.75 second and K_L of 2.65 (parameters selected by $\int_0^\infty T_e^2\,dt$ criterion) are shown in figure 12(a). Overshoot ratio is 1.21 and time to first zero is 0.27 second. Figure 12(b) shows transient responses for τ_c of 0.5 second and K_L of 2.27 (parameters selected by $\int_0^\infty |T_e|\,dt$ criterion). Here overshoot ratio is 1.15 and time to first zero is 0.30 second. Settling time for both of these transients is much shorter than for the transient of figure 10(c). 4212

An analysis of optimum controls for linear feedback control systems was made and is presented in reference 4. One of the control systems considered in this analysis was a speed - fuel-flow control for a turbojet engine. Linearized engine dynamics were represented by a first-order lag term, and the fuel system was considered as having only dead time. The control action of the optimum control for this system was shown to be essentially proportional-plus-integral. Optimization of control-loop parameters was based on minimization of the integral of speed error squared.

The temperature - fuel-flow control system can be considered in the light of this analysis if certain simplifying assumptions are made regarding the system dynamics. An approximation of the frequency-response data presented in figure 6 can be made with a first-order lag term having a time constant of approximately 0.4 second and a dead time of approximately 0.115 second. By using these simplified dynamics, this control system is comparable with the speed - fuel-flow control system of reference 4.

Accordingly, the optimum loop gain should be the system time constant, 0.4 second, divided by the dead time, 0.115 second, or $K_{L,opt} = \frac{0.4}{0.115} = 3.5$. Also, the optimum control integral time constant should equal the system time constant, or $\tau_{c,opt} = 0.4$ second. Comparing these theoretical optimum parameters with the range of parameters (τ_c from 0.75 to 1.0 second and K_L from 2.5 to 3.5) obtained from the experimental integral curves of figure 11(a) shows good agreement in spite of the simplified approximation of the system dynamics.

Rate-limited fuel control. - Since the research-facility fuel control was considerably faster than any practical fuel control now in use on turbojet engines, a control study similar to that discussed in previous sections was made with a rate-limited fuel control. A rate-limiting circuit was inserted between the control and the fuel servo system. This circuit limited the maximum rate of change of fuel flow to approximately 140 percent of rated fuel flow in 0.1 second. Although this rate may be greater than that of a practical fuel control, it was considered sufficiently less than the original rate to show the effects on control performance of a slower fuel control.

Figure 13 shows plots comparing integrals, overshoot ratios, and times to first zero for the original and rate-limited fuel control. Integral criteria values were higher for the rate-limited fuel control, but minimums indicated about the same choice of control integral time constant and loop gain for optimum transient response. The rate-limited fuel control gave slightly greater times to first zero than the original control. At small values of τ_c the rate-limited control gave more temperature overshoot than the original control, and at large values of τ_c the rate-limited control gave less temperature overshoot than the original control. The optimum transient response for the rate-limited fuel control still indicated a satisfactory compromise between overshoot ratio and time to first zero.

<u>Temperature-area control</u>. - Four criteria for evaluating transient temperature response for the temperature-area control system are shown in figure 14. Three of these criteria are the same as used for the temperature - fuel-flow control system. Since there was no temperature overshoot for many of the transients taken with the temperature-area control system, another criterion, time for T_e to reach 10 percent of its initial value, was used instead of time to first zero.

The integral criteria curves of figures 14(a) and (b) show no definite minimums, in contrast with the integral criteria curves for the temperature - fuel-flow control system (fig. 11). These curves indicate that a small τ_c should be used but are not very selective as to loop gain for optimum transient response.

The overshoot-ratio curves of figure 14(c) show a reverse trend to that usually found in control systems. Overshoot ratio decreases with increasing loop gain. These curves are not very helpful in selecting control-loop parameters for optimum transient response other than indicating that loop gain should probably be >3 and $\tau_c \geq 0.25$ second.

The fourth criterion, presented in figure 14(d), also indicates that a small τ_c should be used, at least for loop gains less than 6. At higher loop gains these curves are not very selective as to τ_c. These curves show that the minimum 10-percent response time obtainable with the temperature-area control system was about three times the minimum time to first zero obtainable with the temperature - fuel-flow control system (fig. 9).

In figure 15 are shown transient responses of engine speed, exhaust-valve position, and temperature error for the temperature-area control system. To obtain the desired increase in temperature, the control closed the area valves; but in this case the speed decreased instead of increasing as it did with the temperature - fuel-flow control system (fig. 12).

Figure 15(a) shows the transient responses for τ_c of 0.25 second and K_L of 3.74. This combination of control-loop parameters gave very slight temperature and speed overshoots. The time for T_e to reach 10 percent of its initial value was approximately 0.66 second. The exhaust-valve-position trace shows that the area closed quickly and remained at the limiting position (85 percent of rated area) for approximately 0.4 second and then opened slowly to the steady-state position corresponding to the final desired temperature. 4212

Transient responses for τ_c of 0.25 second and K_L of 9.96 are shown in figure 15(b). There was no temperature overshoot and very slight speed overshoot. The time for T_e to reach 10 percent of its initial value was approximately 0.63 second. During this transient, the exhaust valves remained at the limiting position for approximately 0.58 second and then very quickly opened to the final steady-state position.

The 10-percent response times obtained from the transient data of figure 15 do not clearly indicate a choice of loop gain; but, considering the area traces, it would be preferable to use the lower value of loop gain. At the higher loop gain the area valve is more sensitive to the noise in the temperature signal.

CONCLUDING REMARKS

In the high-speed operating region, tailpipe gas temperature could be satisfactorily controlled during small transients by using temperature feedback control systems which varied either fuel flow or exhaust-nozzle area. A proportional-plus-integral control in a temperature - fuel-flow control system provided satisfactory transient response to a desired step increase in temperature. For a temperature-area control system, it was necessary to add nonlinear components to the basic proportional-plus-integral control to provide satisfactory temperature response during transients.

For the essentially linear temperature - fuel-flow control system, error function integrals such as $\int_0^\infty T_e^2\,dt$ and $\int_0^\infty |T_e|\,dt$ (where T_e is the temperature error and t is time) appeared to be better criteria than overshoot ratio and time to first zero for selecting control-loop parameters for optimum transient temperature response. Control-loop parameters selected on the basis of these integral criteria agreed very well with optimum predictions of a previous analysis.

Limiting the rate of change of fuel flow did not appreciably alter the selection of control-loop parameters for optimum transient temperature response.

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For the temperature-area control system, which is nonlinear because of area saturation limits, the integral criteria, the overshoot ratio, and the time for T_e to reach 10 percent of its initial value were somewhat inadequate for selecting loop gain for optimum transient temperature response. However, these criteria did indicate that a small control integral time constant should be used.

The transfer functions given by equations (1) and (2) provided a satisfactory description of engine and instrumentation dynamics in the high-speed operating region.

Lewis Flight Propulsion Laboratory
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APPENDIX A

SYMBOLS

A	exhaust-nozzle area, sq in.
a	initial rise ratio of temperature to step change in fuel flow at constant area 4212
b	initial rise ratio of temperature to step change in area at constant fuel flow
E	voltage
$G(s)$	transfer function
K_L	loop gain, product of frequency invariant gains around loop
$K_{L,opt}$	optimum loop gain
N	engine speed, rpm
s	complex variable used in Laplace transformation methods
T_e	error between indicated and reference temperatures, $^{\circ}F$
T_i	indicated tailpipe gas temperature, $^{\circ}F$
t	time, sec
$t_{d,a}$	dead time of temperature or speed to area, sec
$t_{d,f}$	dead time of temperature or speed to fuel flow, sec
w_f	indicated fuel flow ($\propto$ fuel-valve position), lb/hr
τ_c	control integral time constant, sec
$\tau_{c,opt}$	optimum control integral time constant, sec
τ_e	engine time constant, sec
τ_t	thermocouple time constant, sec
τ_1	time constant of a lead term of engine dynamics, sec
τ_2	time constant of a lag term of engine dynamics, sec

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APPENDIX B

METHOD OF OBTAINING ENGINE DYNAMICS FROM

FREQUENCY-RESPONSE DATA

Graphical methods can be applied to the speed and temperature frequency-response data to evaluate the various time constants, dead times, and initial rise ratios of equations (1) and (2). These methods, which will be described in this appendix, are based upon the manipulation of various transfer functions. Derivation of transfer functions describing engine dynamic behavior can be found in reference 5.

The engine time constant can be obtained from the speed-area frequency-response data shown in figure 16. The normalized transfer function of engine speed to exhaust-nozzle area at constant fuel flow can be represented by

$$\left.\frac{\Delta N}{\Delta A}(s)\right|_{w_f} = \frac{e^{-t_{d,a}s}}{1 + \tau_e s} \tag{B1}$$

By fitting a first-order response curve to the experimental data in figure 16(a), an engine time constant of 0.83 second was determined. Also, by considering the difference between the experimental data and the first-order phase-shift curve in figure 16(b), a speed-area dead time of 0.021 second was determined.

The normalized transfer function of engine speed to indicated fuel flow at constant area can be represented by

$$\left.\frac{\Delta N}{\Delta w_f}(s)\right|_{A} = \frac{G(s)e^{-t_{d,f}s}}{1 + \tau_e s} \tag{B2}$$

Experimental data representing the speed - fuel-flow frequency response are shown in figure 17. Also shown in this figure are calculated first-order response curves of $\frac{1}{1 + \tau_e s}$ with $\tau_e = 0.83$ second. The difference between these response curves and the experimental data represents $G(s)e^{-t_{d,f}s}$, the fuel-distribution-system and combustion dynamics. Plots of these dynamics are shown in figure 18.

The normalized transfer function of indicated tailpipe gas temperature to indicated fuel flow at constant area can be represented by

$$\frac{\Delta T_i}{\Delta w_f}(s)\bigg|_A = \frac{(1 + a\tau_e s)(1 + \tau_1 s)G(s)e^{-t_{d,f}s}}{(1 + \tau_e s)(1 + \tau_2 s)(1 + \tau_t s)} \tag{1}$$

Dividing equation (1) by equation (B2) results in the following equation:

$$\frac{\Delta T_i}{\Delta N}(s)\bigg|_A = \frac{(1 + a\tau_e s)(1 + \tau_1 s)}{(1 + \tau_t s)(1 + \tau_2 s)} \tag{B3}$$

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Data corresponding to equation (B3) can be calculated from the experimental data of figures 6 and 17. Plots of these calculated data are shown in figure 19. Also shown in figure 19 are calculated frequency-response curves representing equation (B3). Numerical values for the rise ratio and time constants were chosen to make the response curves fit the data with reasonable accuracy. Thus values for initial rise ratio a of 1.3 and for thermocouple time constant τ_t of 0.3 second were determined. Time constants of 9 and 10 seconds were used for the other lead and lag terms of the engine dynamics.

The normalized transfer function of indicated tailpipe gas temperature to exhaust-nozzle area at constant fuel flow can be represented by

$$\frac{\Delta T_i}{\Delta A}(s)\bigg|_{w_f} = \frac{(1 + b\tau_e s)(1 + \tau_1 s)e^{-t_{d,a}s}}{(1 + \tau_e s)(1 + \tau_2 s)(1 + \tau_t s)} \tag{2}$$

The dead time and all the time constants of this transfer function have been determined by the methods described in the preceding paragraphs. Thus, by extracting the product of all the terms except $1 + b\tau_e s$ from the data corresponding to this transfer function (fig. 7), plots of $1 + b\tau_e s$ were obtained. These plots are shown in figure 20. By fitting a first-order response curve to these calculated data, a value for the initial rise ratio b of approximately 0.4 was determined.

The numerical values of the time constants, dead time, and initial rise ratios determined by the methods described in this appendix are valid in the range of operation where the frequency-response data were obtained. Since these values vary with operating range, a more complete description of engine dynamics could be obtained from frequency-response data taken at several operating points.

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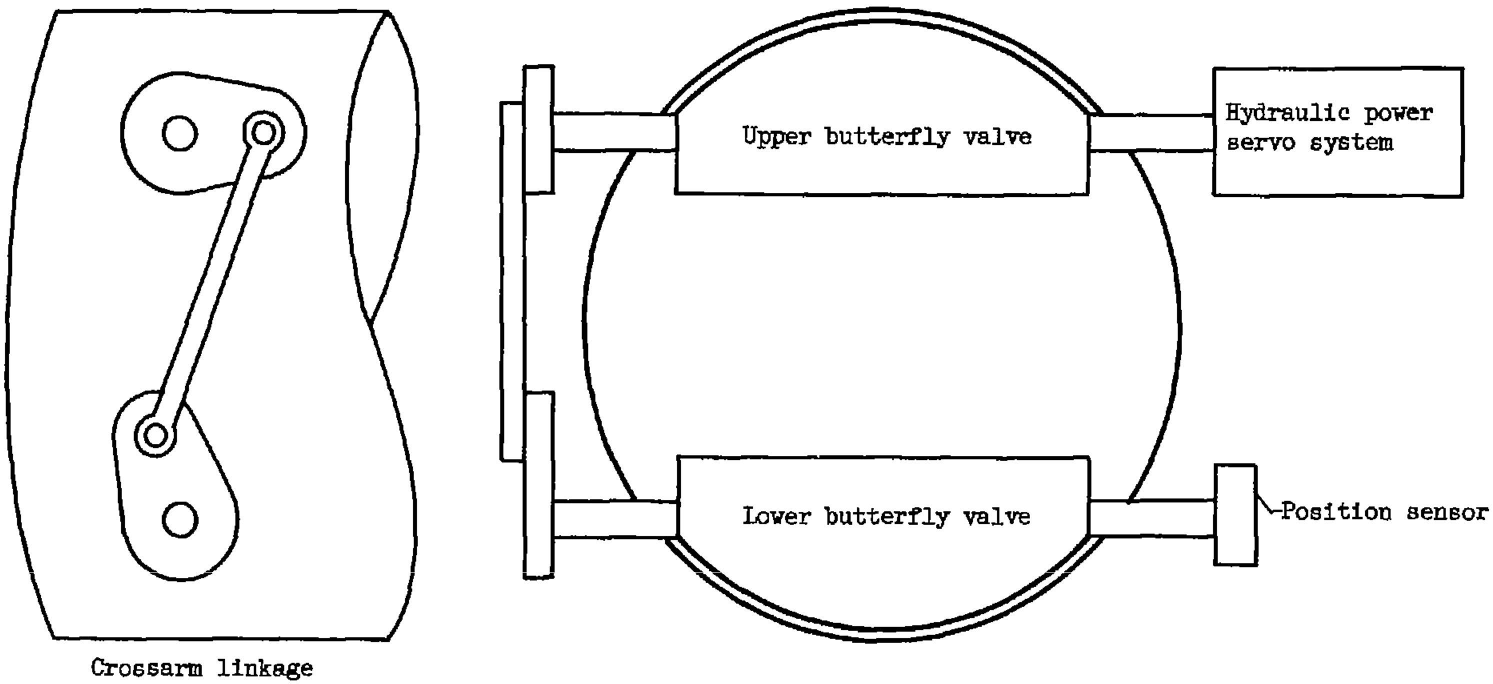
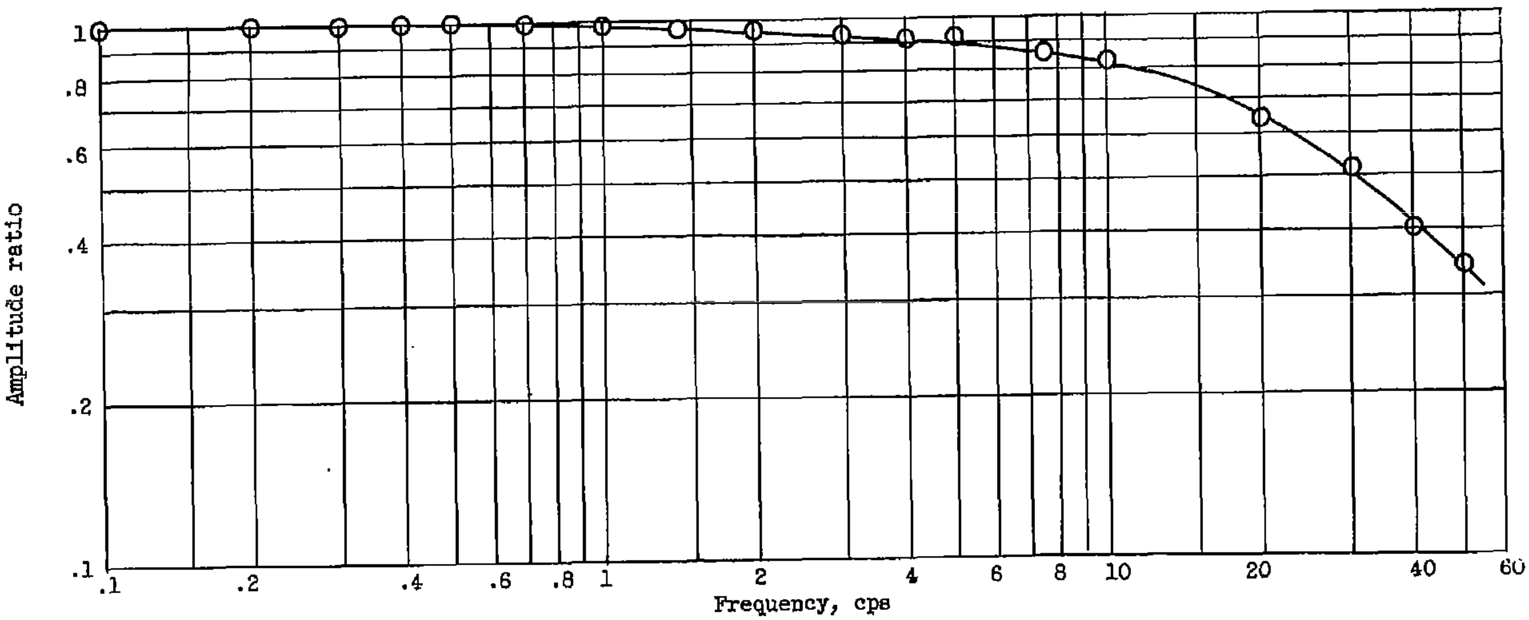
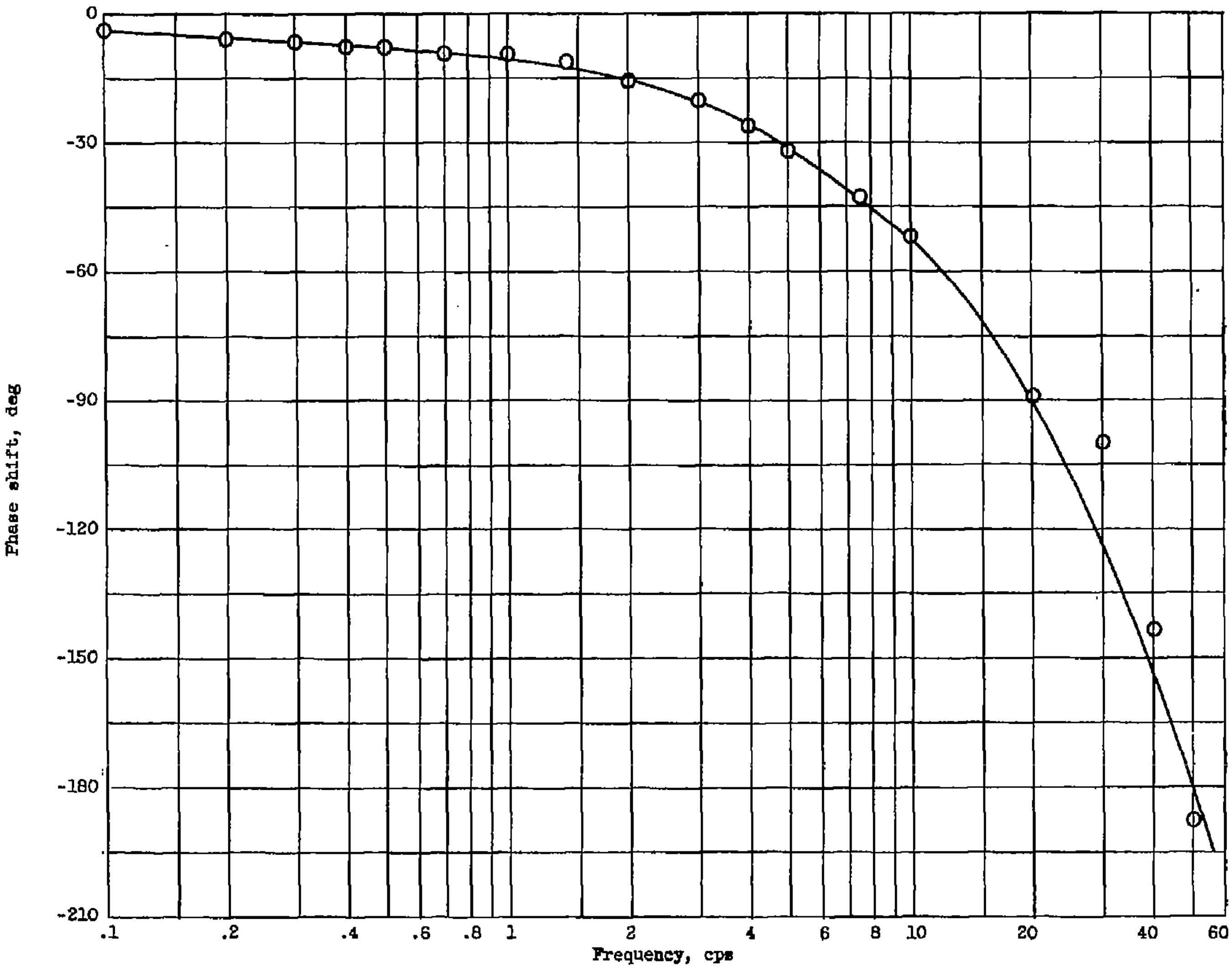


Figure 1. - Schematic of variable-area exhaust-nozzle system.



(a) Amplitude ratio.

Figure 2. - Frequency response of exhaust-valve position to sinusoidal input signal. Area variation, ±5 percent of rated area; operating point, 100 percent of rated area.



(b) Phase shift.

Figure 2. - Concluded. Frequency response of exhaust-valve position to sinusoidal input signal. Area variation, ±5 percent of rated area; operating point, 100 percent of rated area.

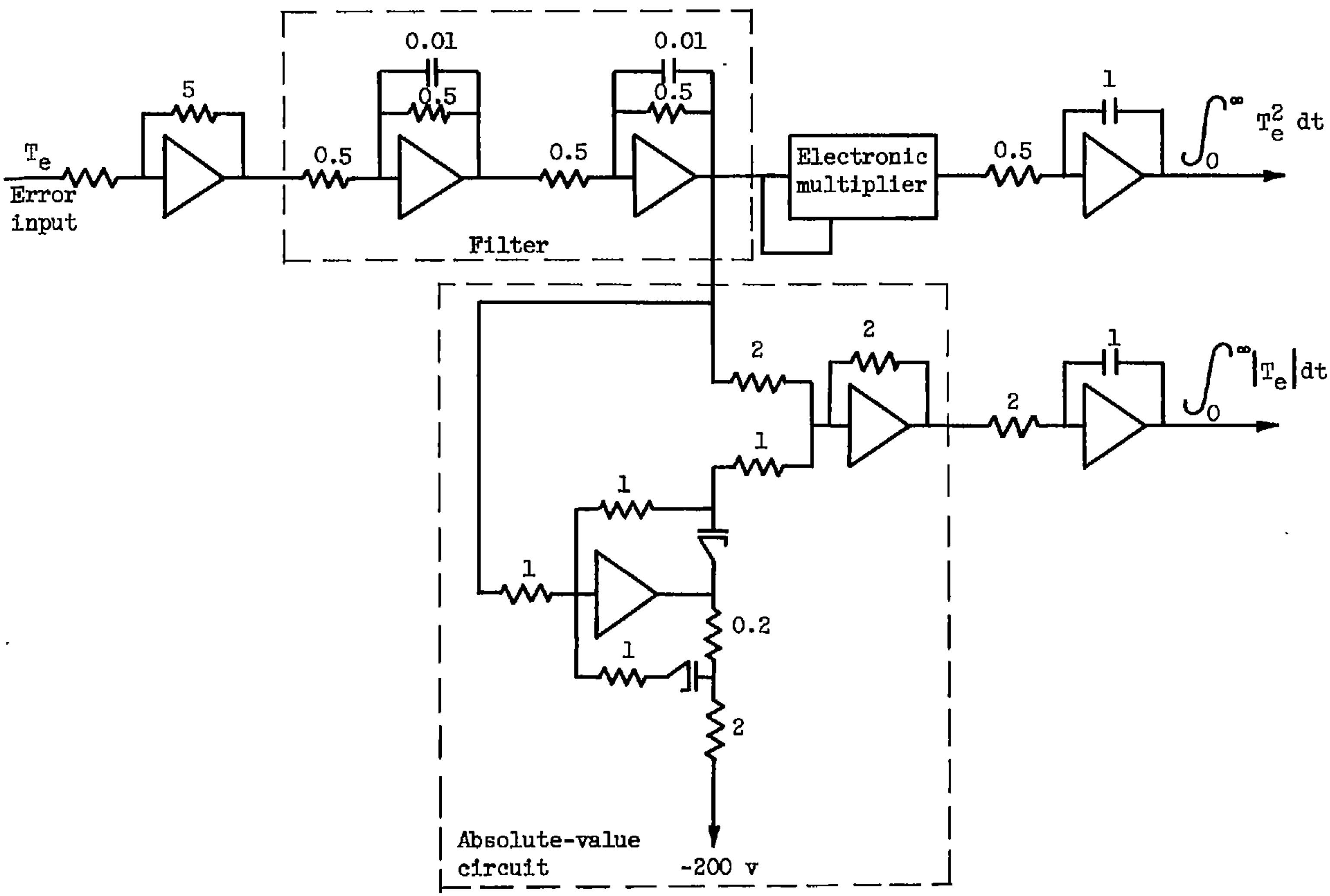


Figure 3. - Diagram of computer circuit used to obtain error integrals. Resistance in megohms, capacitance in microfarads.

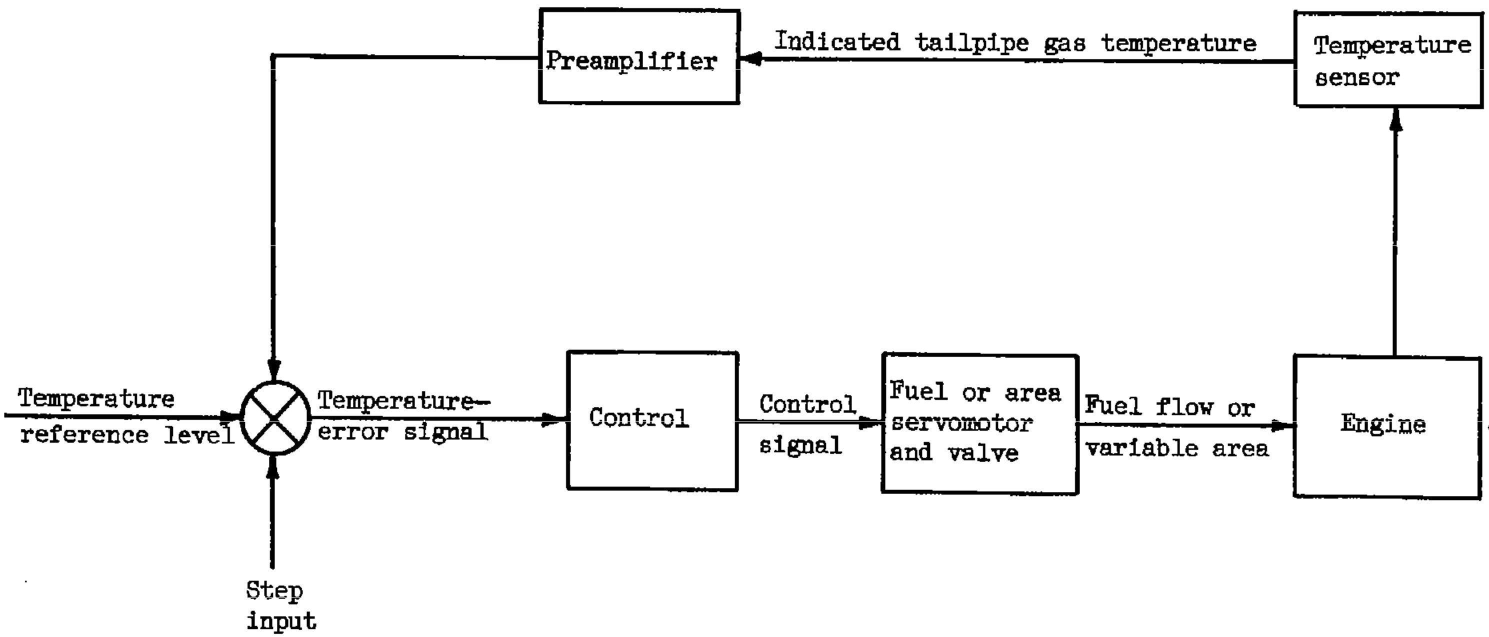
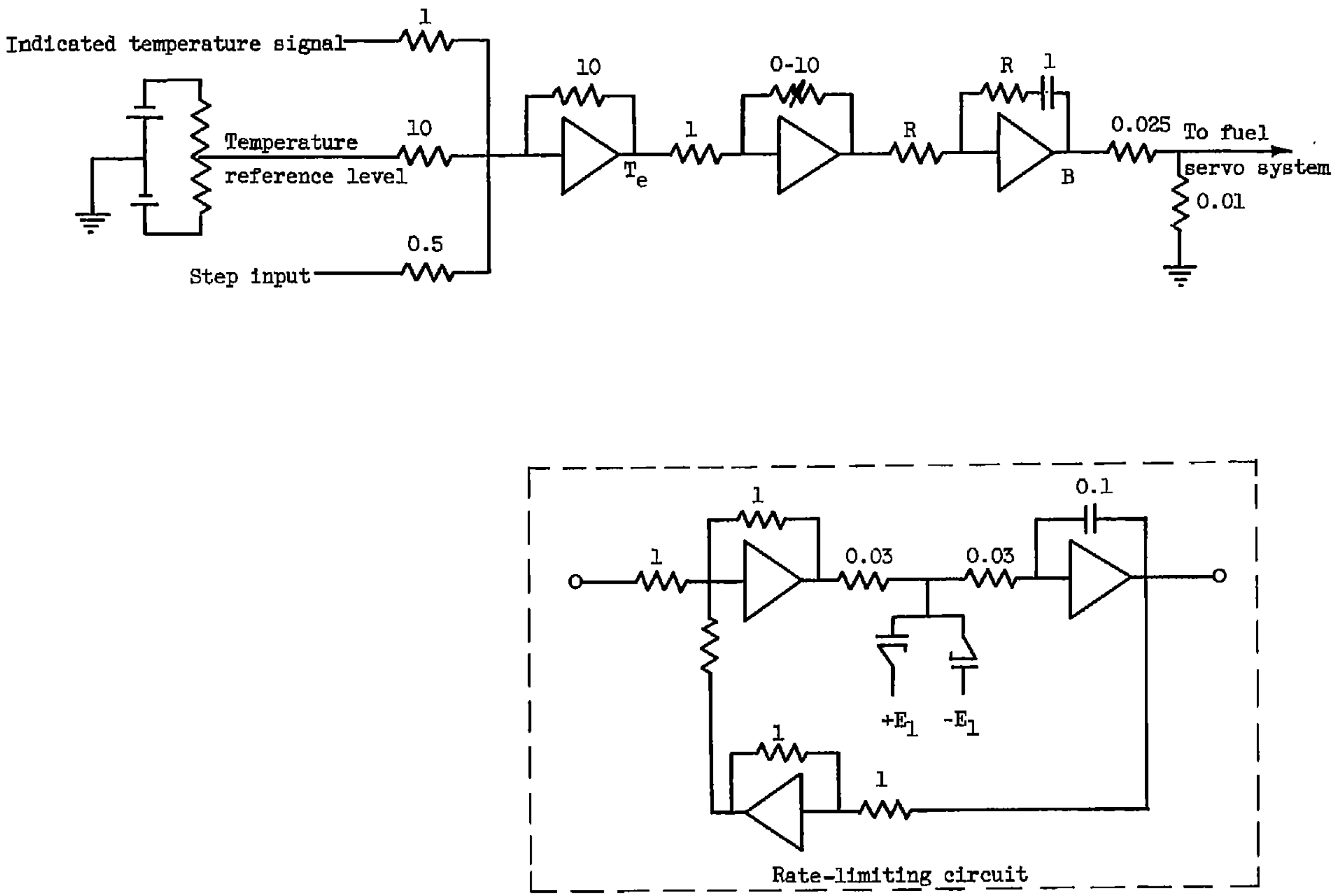
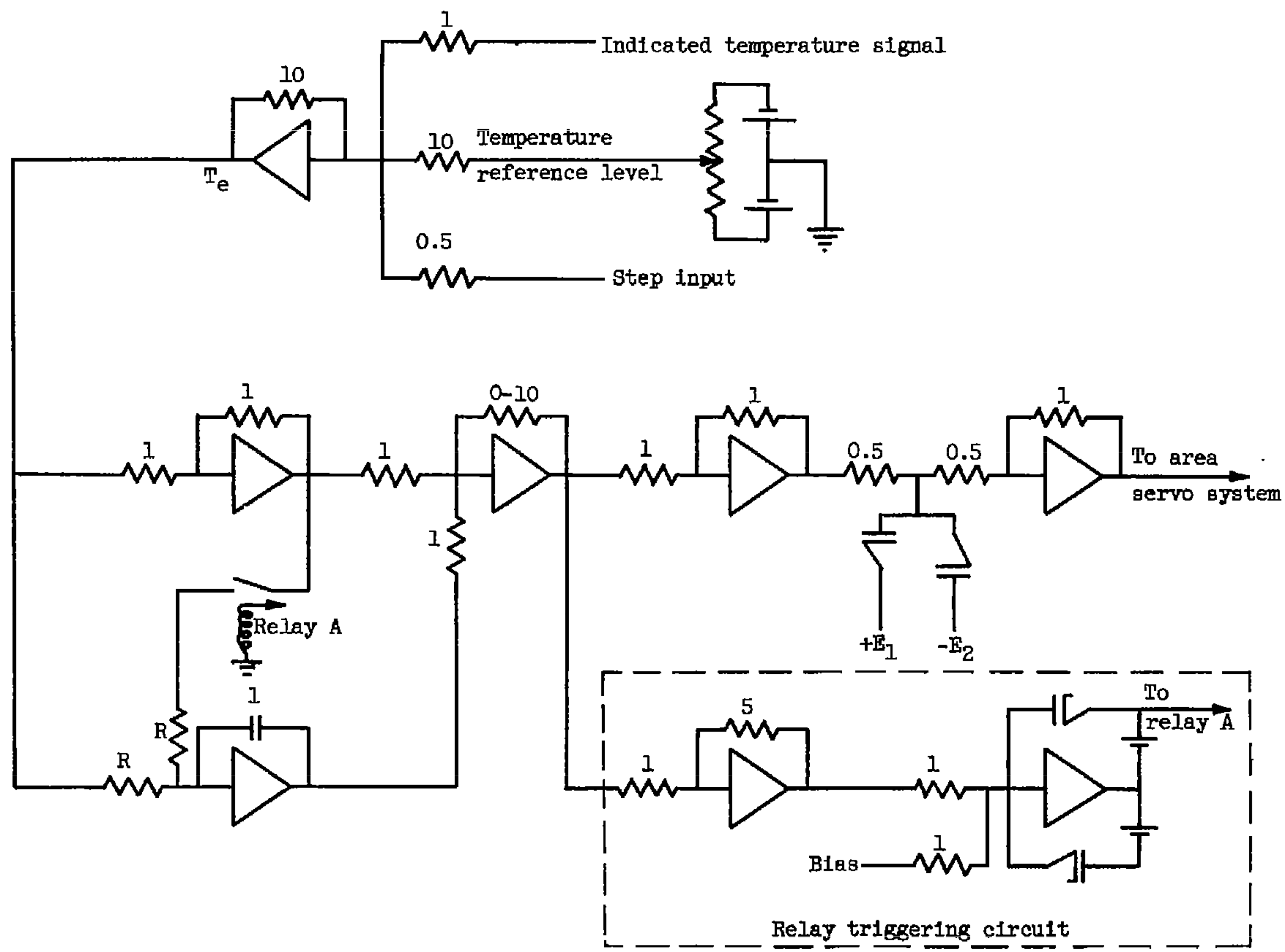


Figure 4. - Block diagram of temperature feedback control system.



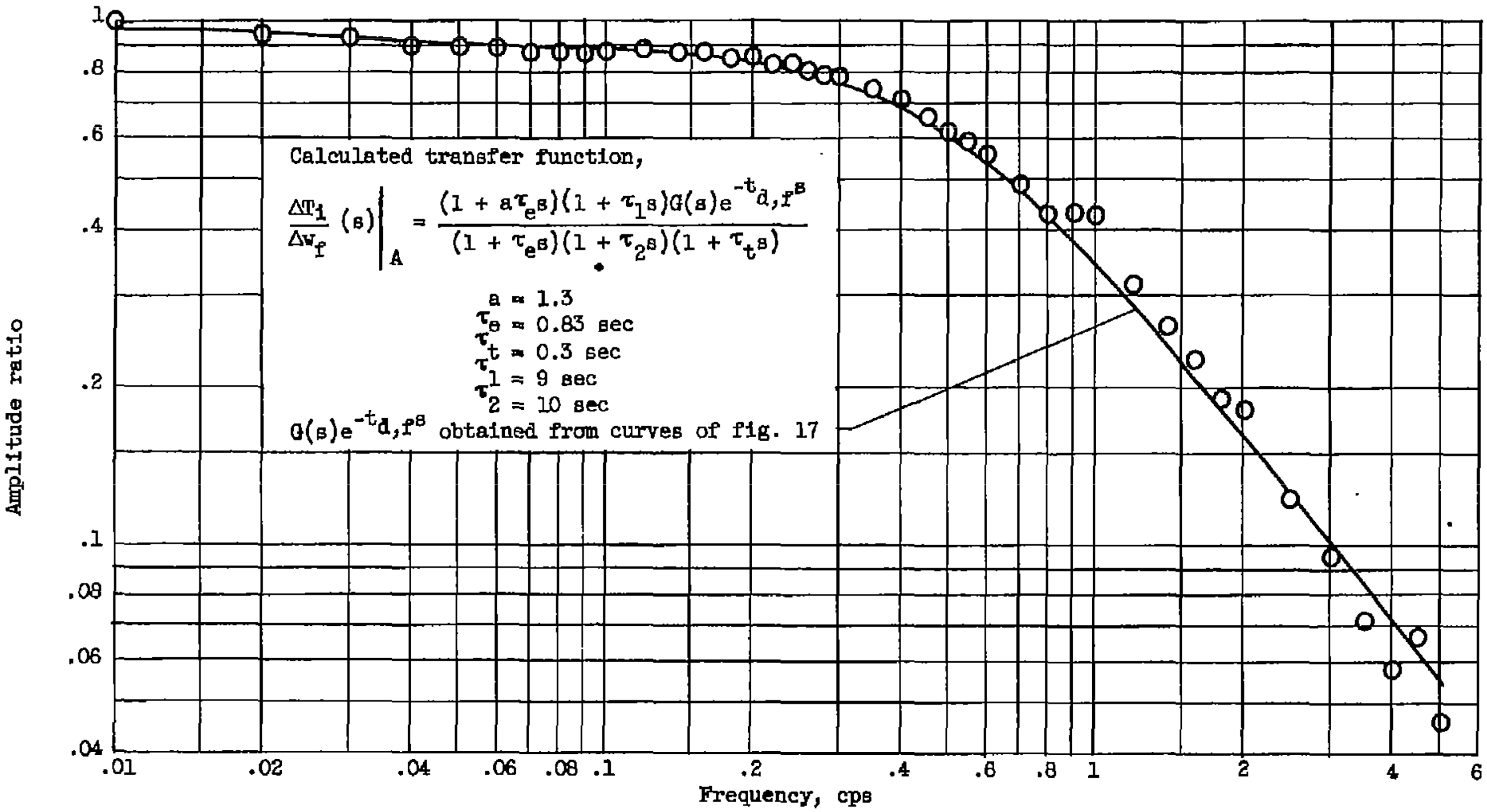
(a) Temperature - fuel-flow control system.

Figure 5. - Computer circuit diagram of summing network and control. Resistance in megohms, capacitance in microfarads.



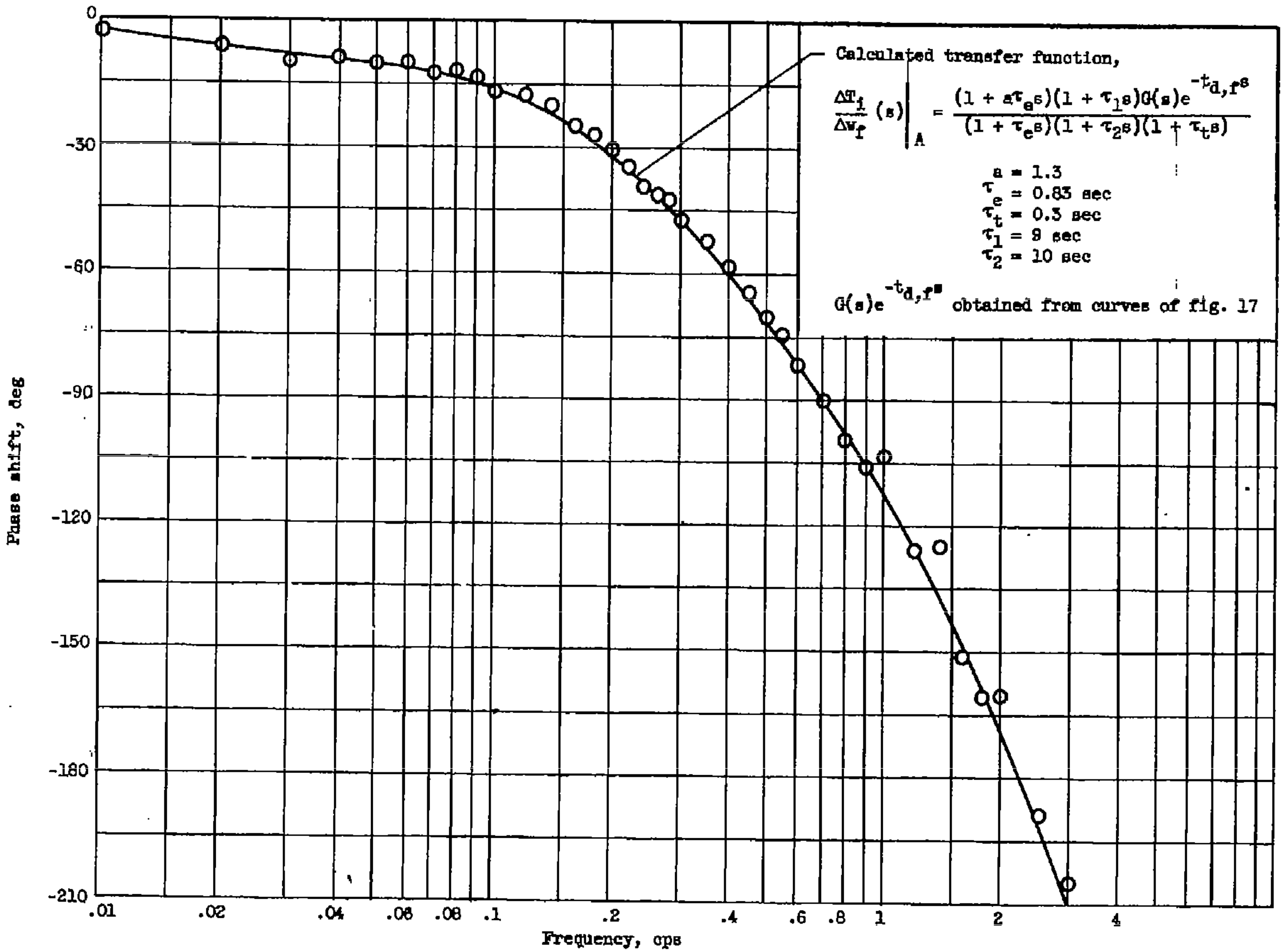
(b) Temperature-area control system.

Figure 5. - Concluded. Computer circuit diagram of summing network and control. Resistance in megohms, capacitance in microfarads.



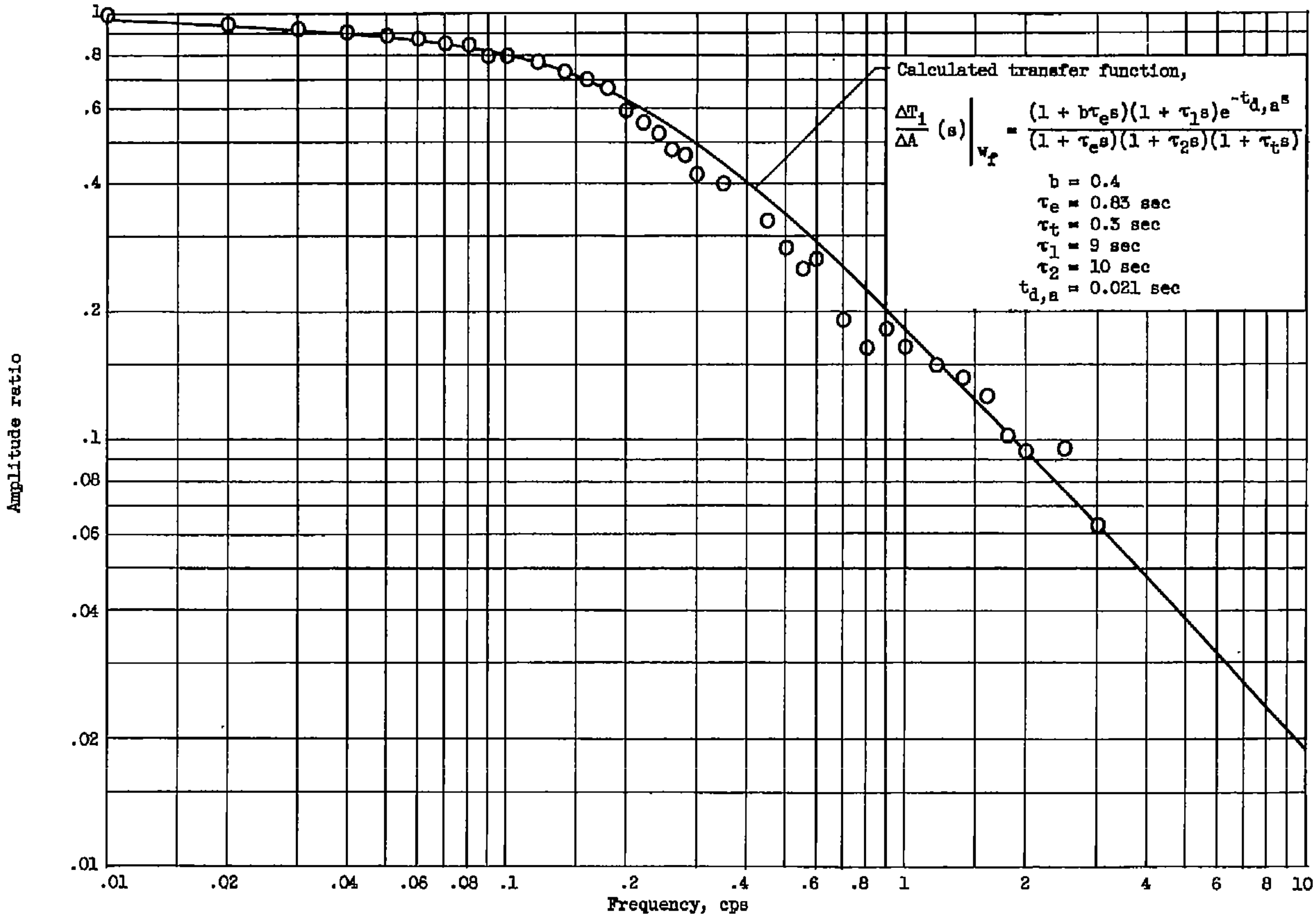
(a) Amplitude ratio.

Figure 6. - Frequency response of indicated tailpipe gas temperature to indicated fuel flow at constant area. Temperature variation, ±5.5 percent of rated temperature; operating point, 79 percent of rated temperature.



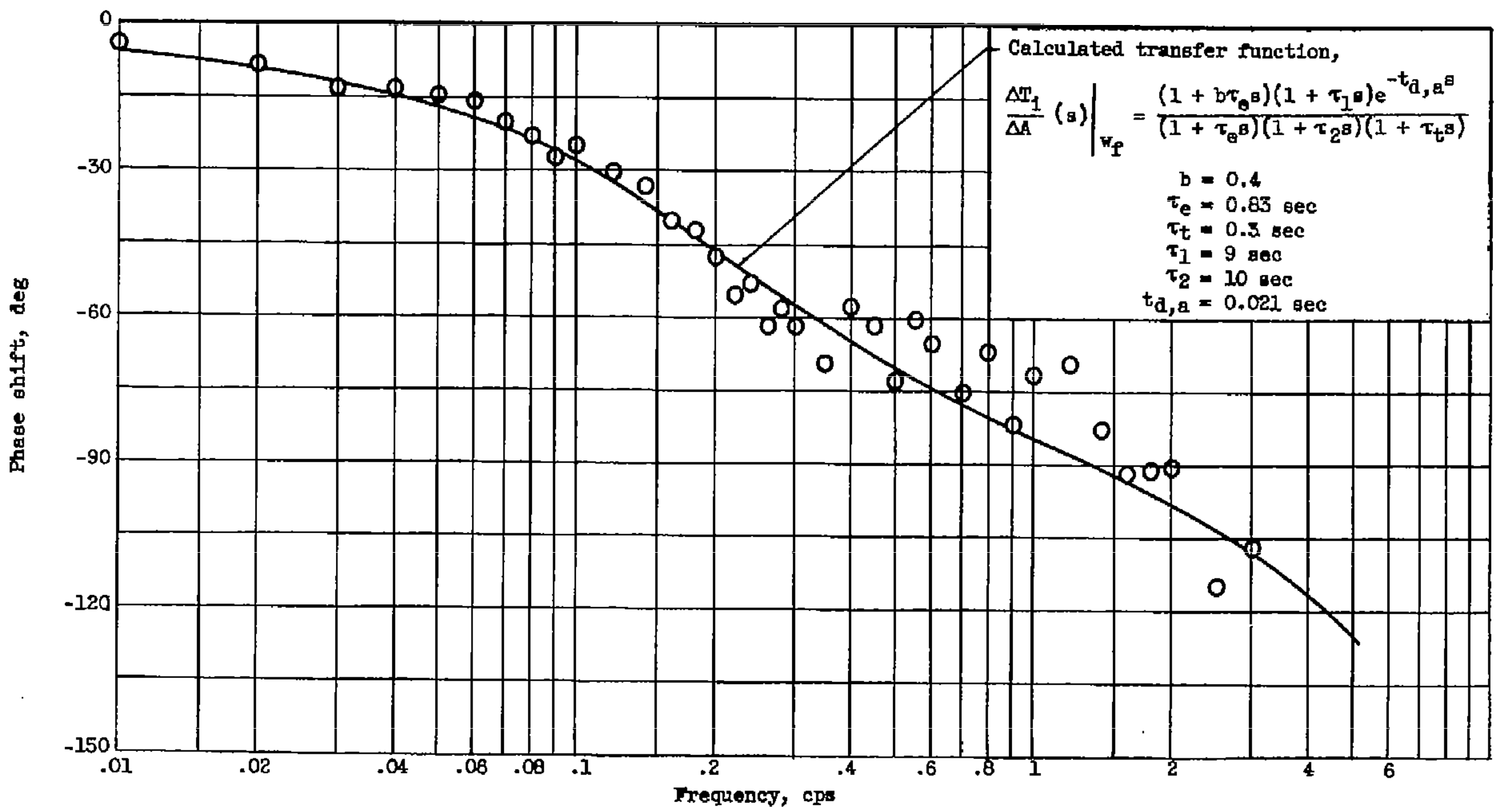
(b) Phase shift.

Figure 6. - Concluded. Frequency response of indicated tailpipe gas temperature to indicated fuel flow at constant area. Temperature variation, ±5.5 percent of rated temperature; operating point, 79 percent of rated temperature.



(a) Amplitude ratio.

Figure 7. - Frequency response of indicated tailpipe gas temperature to exhaust-nozzle area at constant fuel flow. Temperature variation, ±2.7 percent of rated temperature; operating point, 78.5 percent of rated temperature.



(b) Phase shift.

Figure 7. - Concluded. Frequency response of indicated tailpipe gas temperature to exhaust-nozzle area at constant fuel flow. Temperature variation, ±2.7 percent of rated temperature; operating point, 78.5 percent of rated temperature.

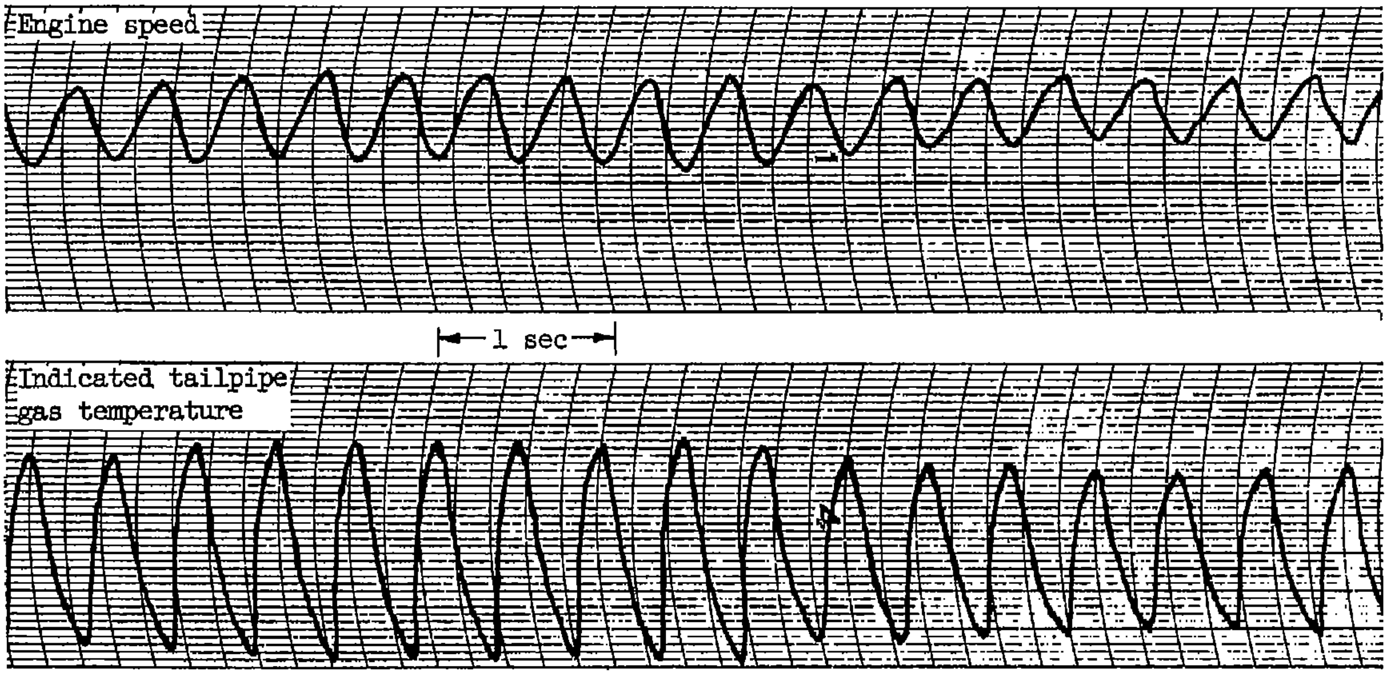


Figure 8. - Stability-limit check for temperature - fuel-flow control. Loop gain, 6.3; control integral time constant, 0.5 second.

4212

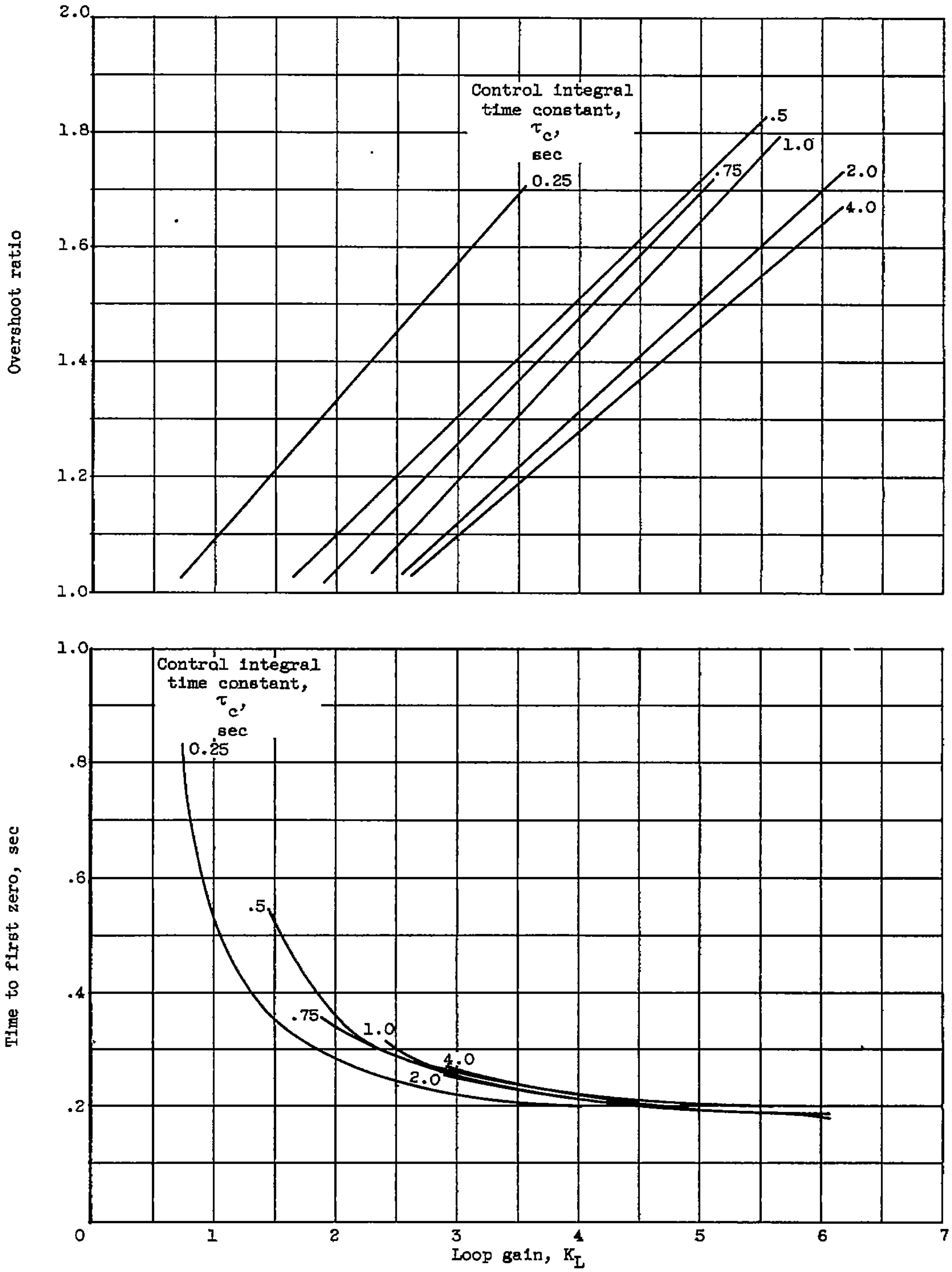
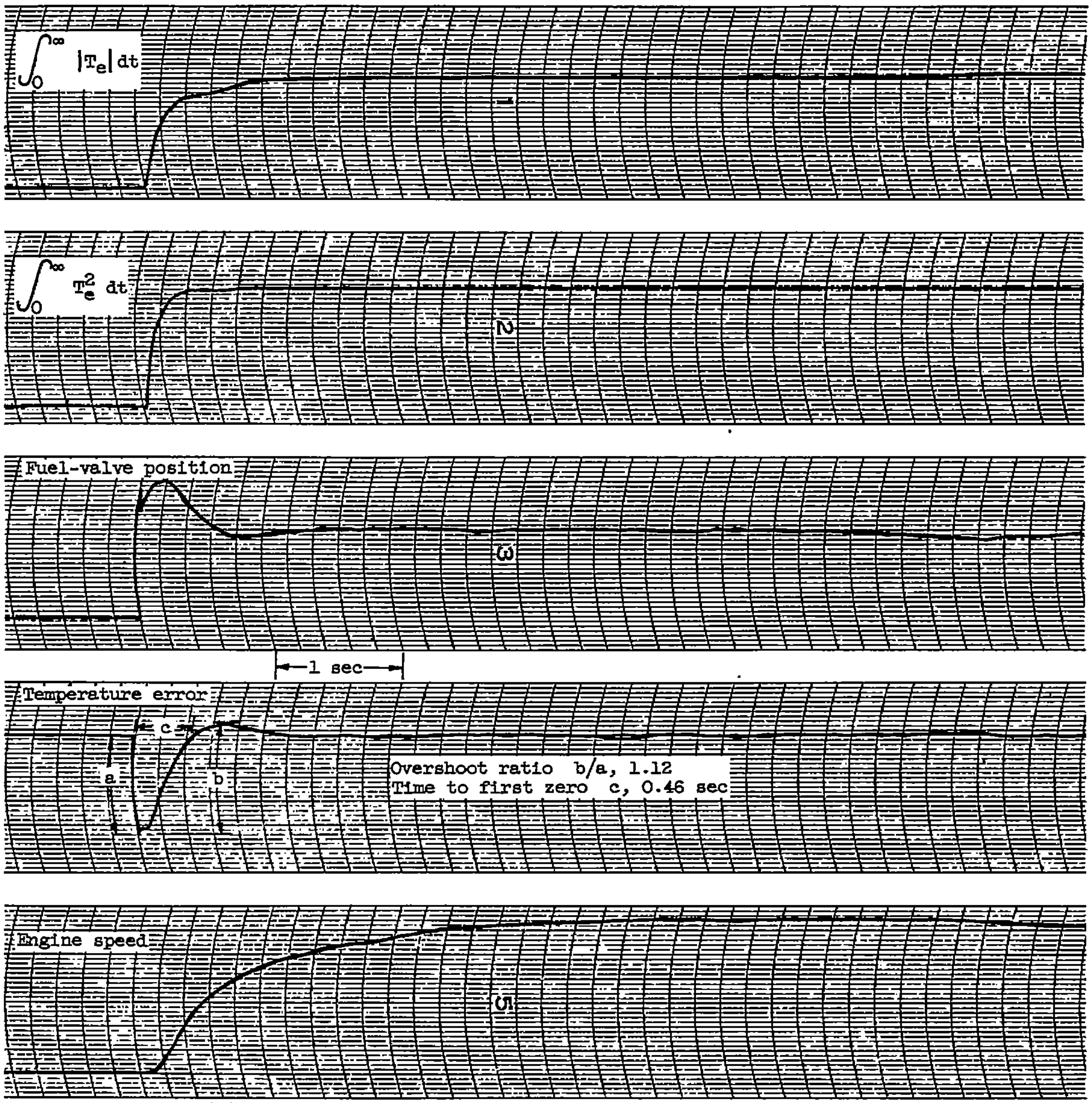


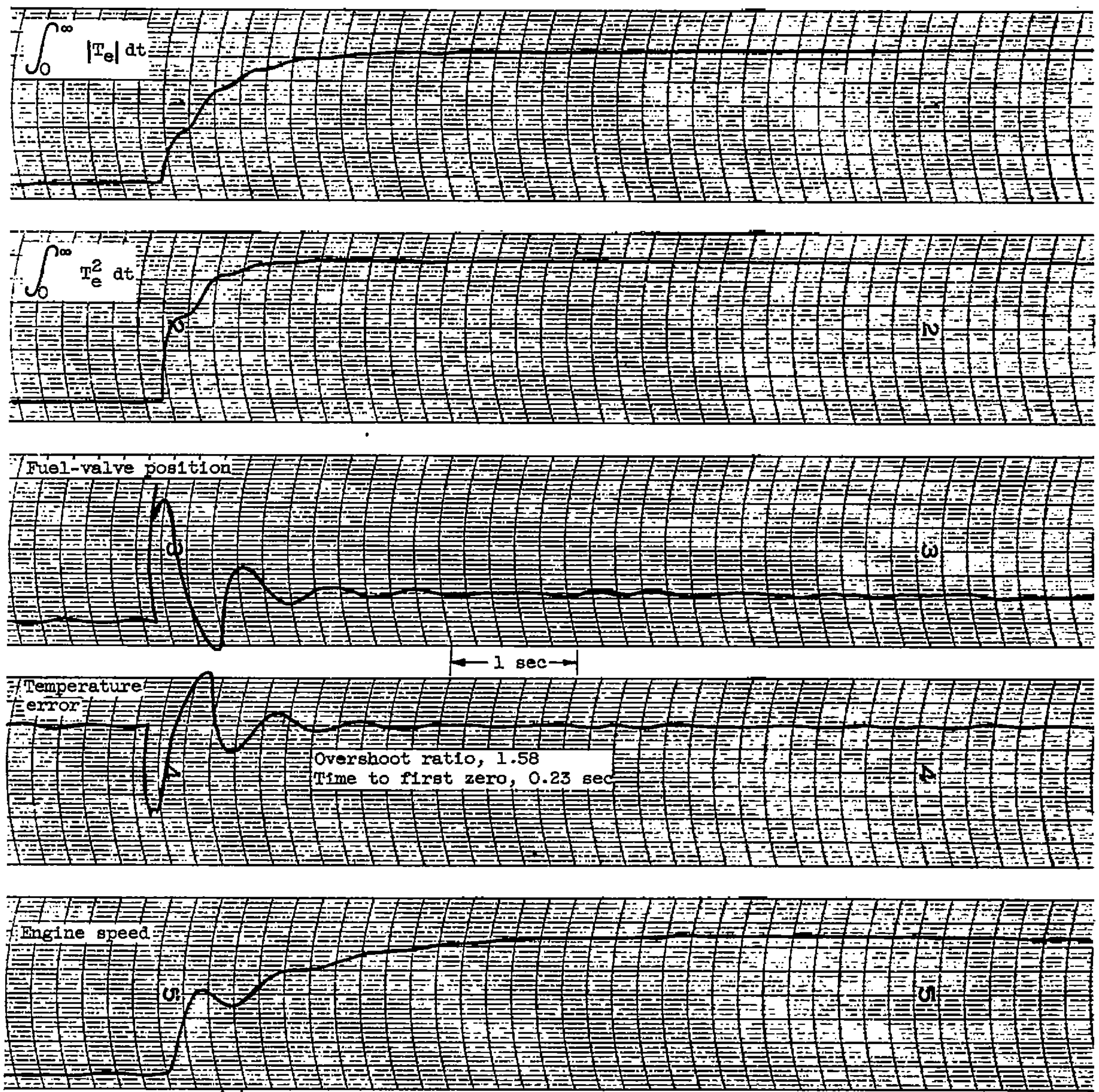
Figure 9. - Criteria for evaluating transient temperature response for temperature - fuel-flow control.

4212



(a) Control integral time constant, 0.25 second; loop gain, 1.14.

Figure 10. - Transient responses to step increase in reference temperature for temperature - fuel-flow control system.

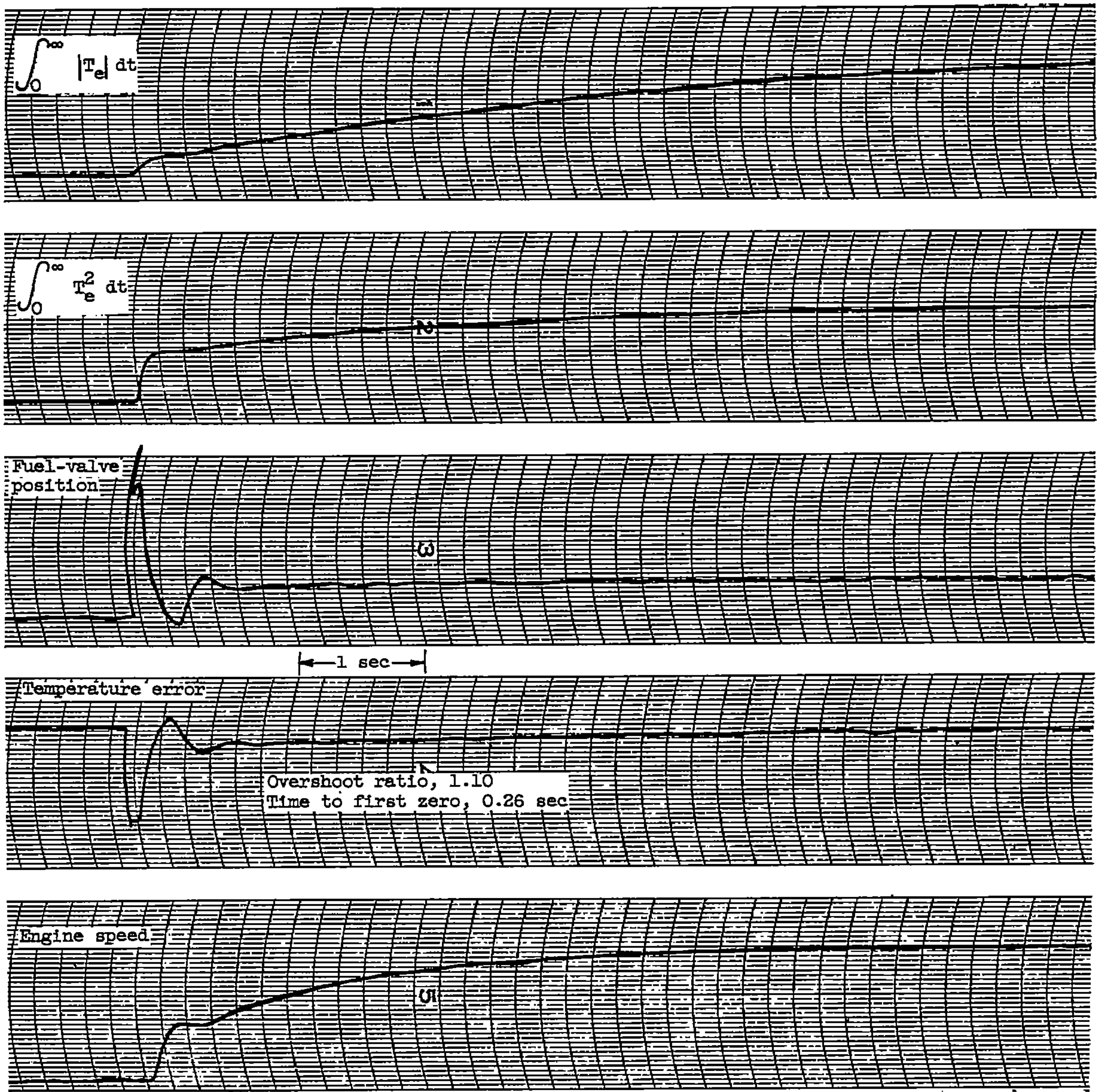


(b) Control integral time constant, 0.25 second; loop gain, 3.03.

Figure 10. - Continued. Transient responses to step increase in reference temperature for temperature - fuel-flow control system.

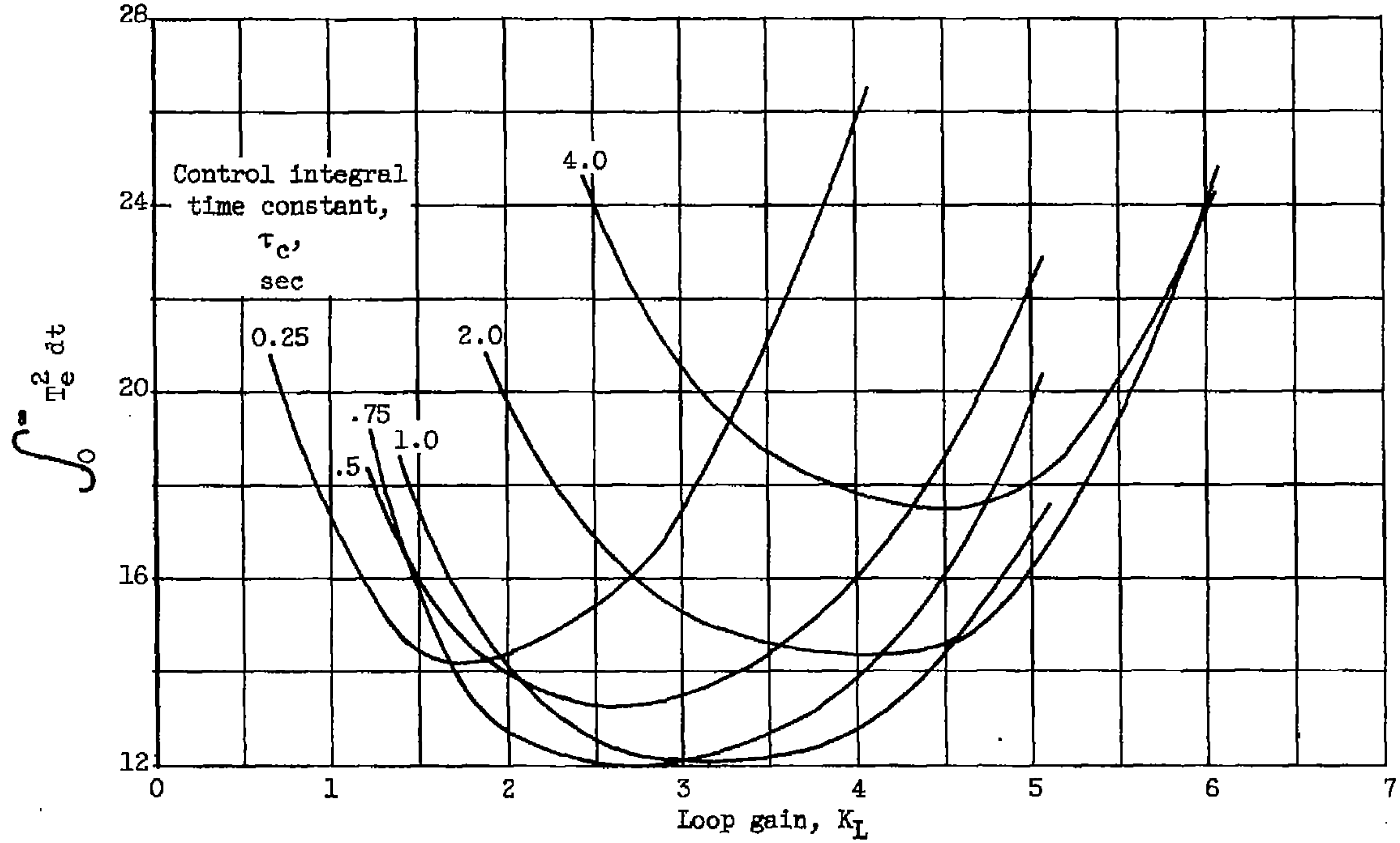
4212

CE-5



(c) Control integral time constant, 4.0 seconds; loop gain, 3.03.

Figure 10. - Concluded. Transient responses to step increase in reference temperature for temperature - fuel-flow control system.

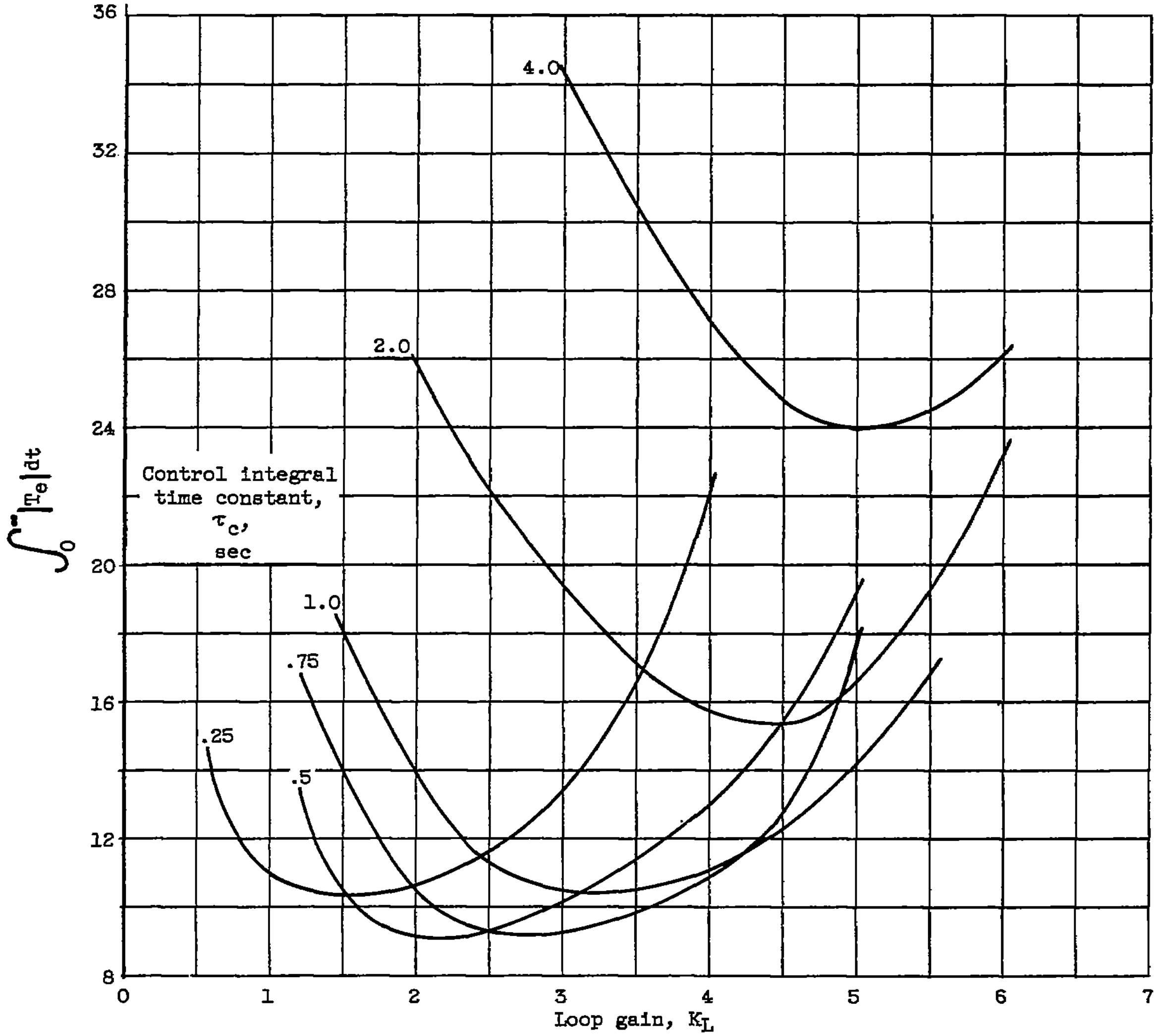


(a) $\int_0^\infty T_e^2\,dt$.

Figure 11. - Criteria for evaluating transient temperature response for temperature - fuel-flow control system.

4212

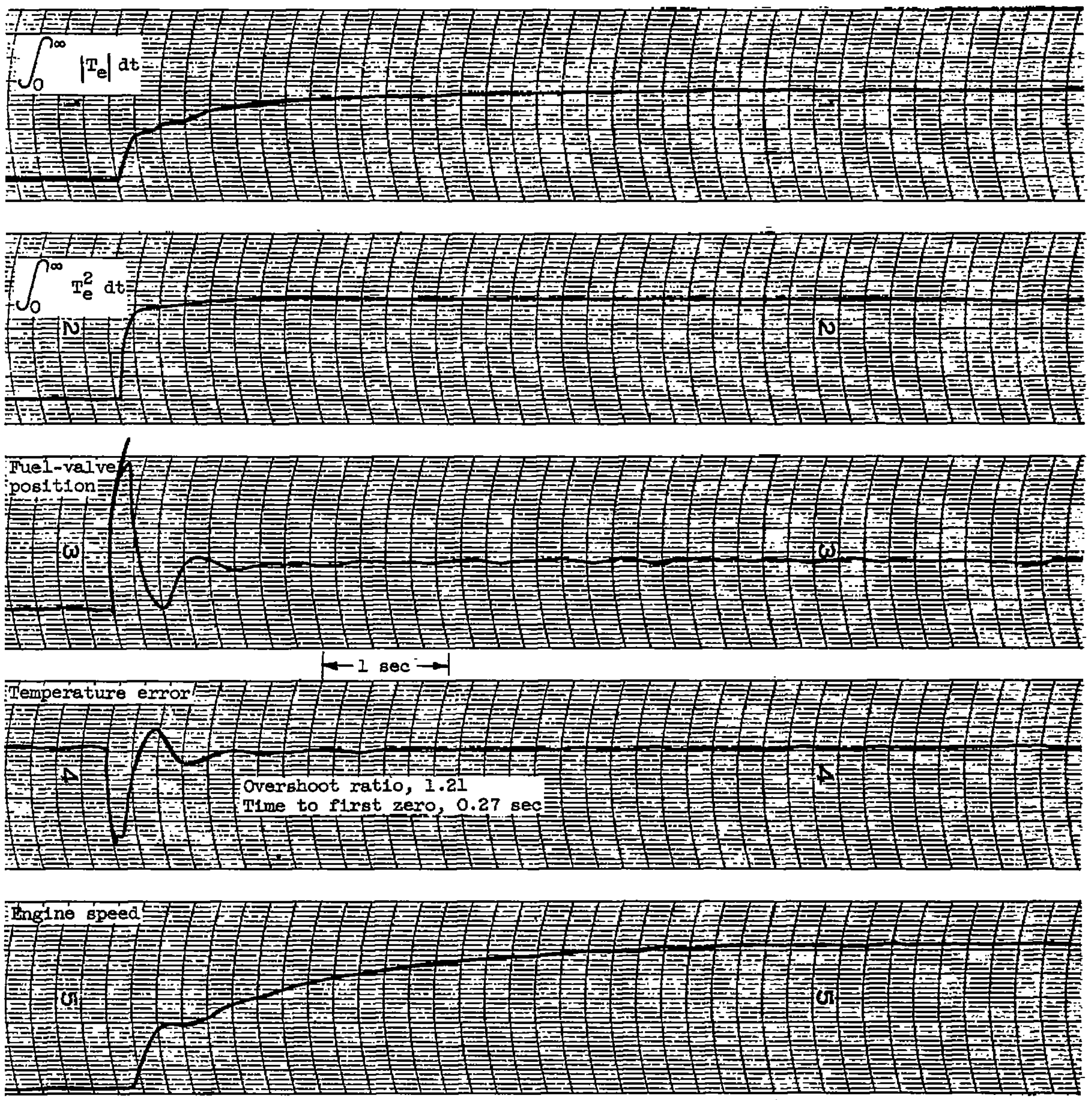
CE-5 back



(b) $\int_0^\infty |T_e| dt$.

Figure 11. - Concluded. Criteria for evaluating transient temperature response for temperature - fuel-flow control system.

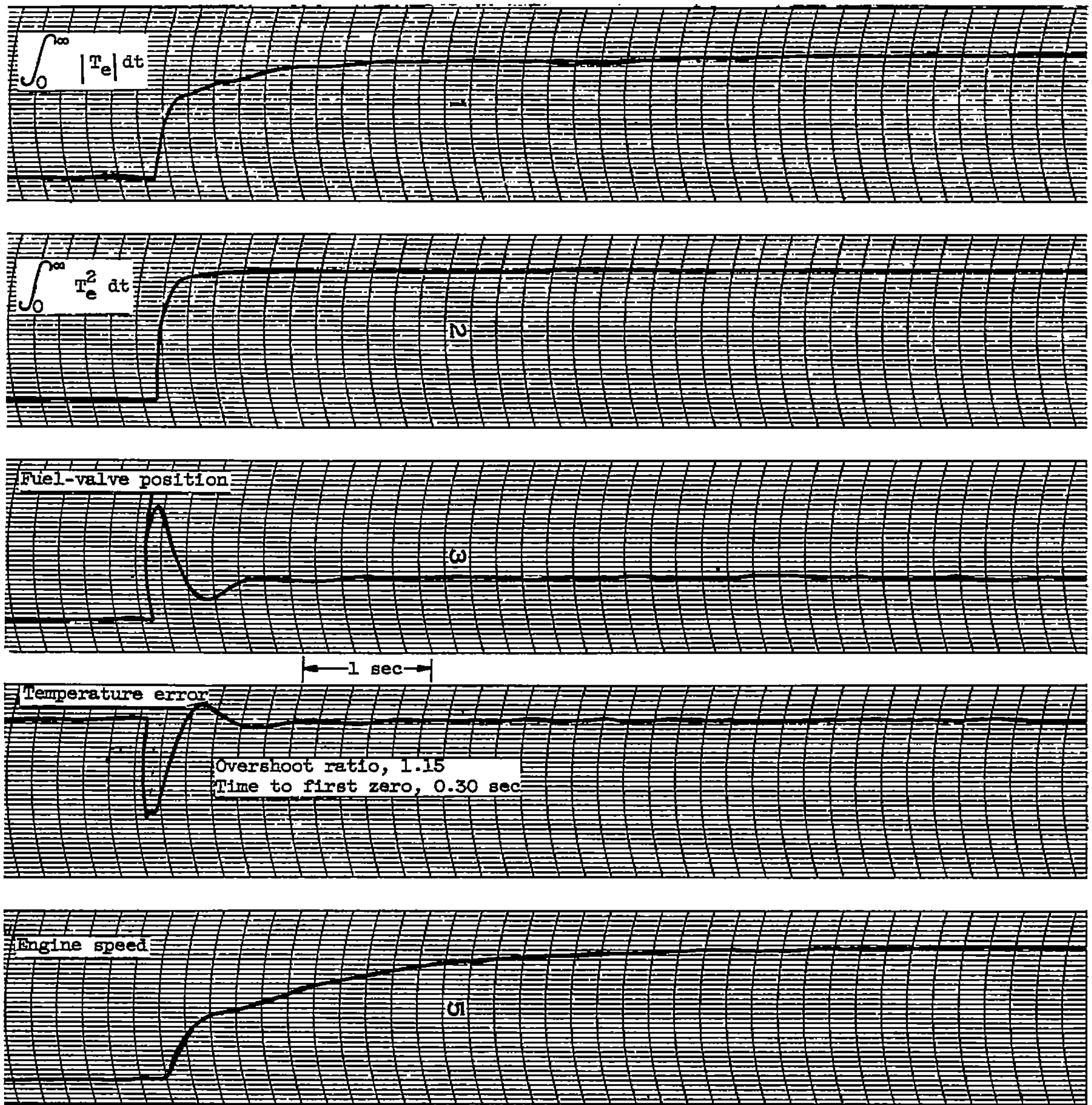
4212



(a) Control-integral time constant, 0.75 second; loop gain, 2.65.

Figure 12. - Transient responses to step increase in reference temperature for temperature - fuel-flow control system.

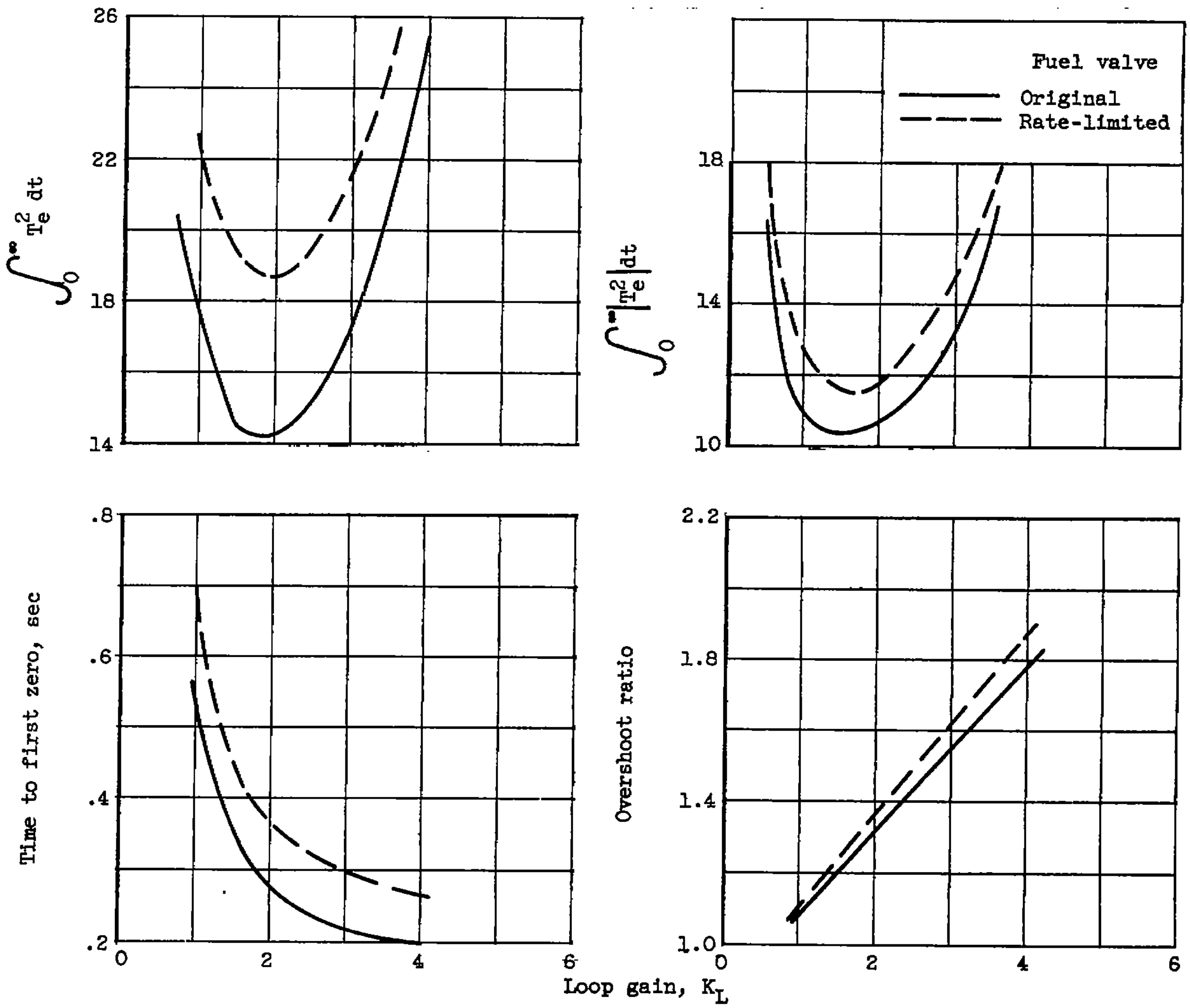
4212



(b) Control integral time constant, 0.5 second; loop gain, 2.27.

Figure 12. - Concluded. Transient responses to step increase in reference temperature for temperature - fuel-flow control system.

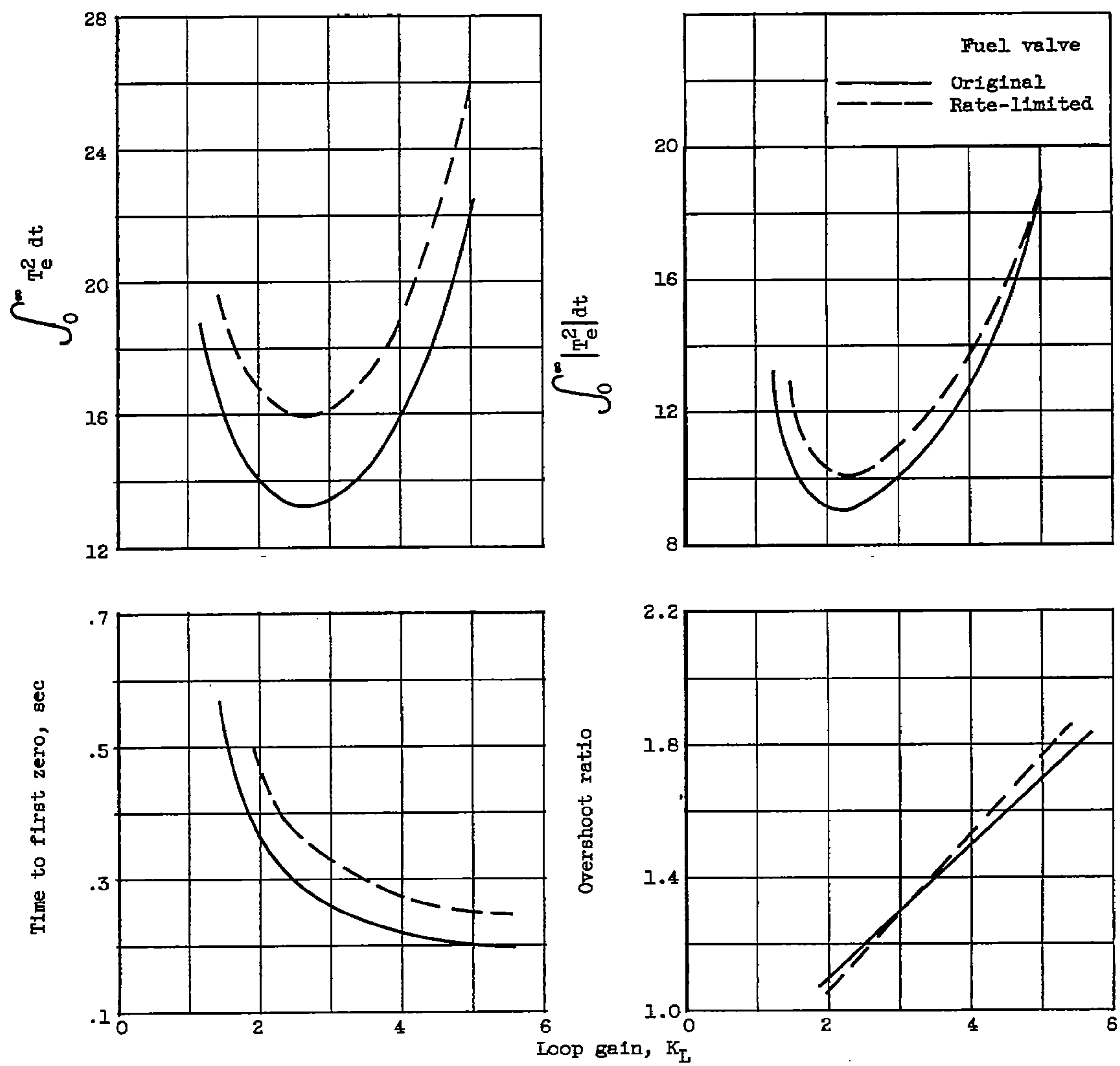
4212



(a) Control integral time constant, 0.25 second.

Figure 13. - Comparison of transient-temperature-response criteria for rate-limited and original fuel-flow control systems.

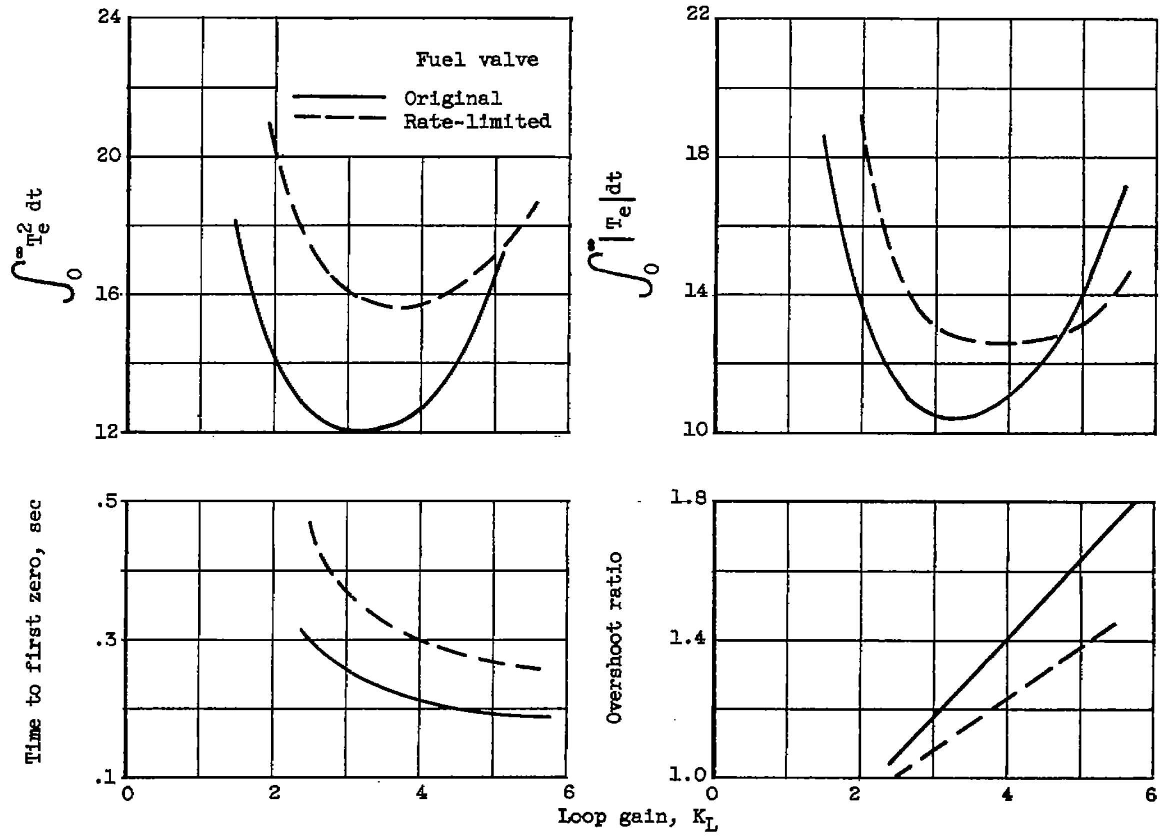
4212



(b) Control integral time constant, 0.5 second.

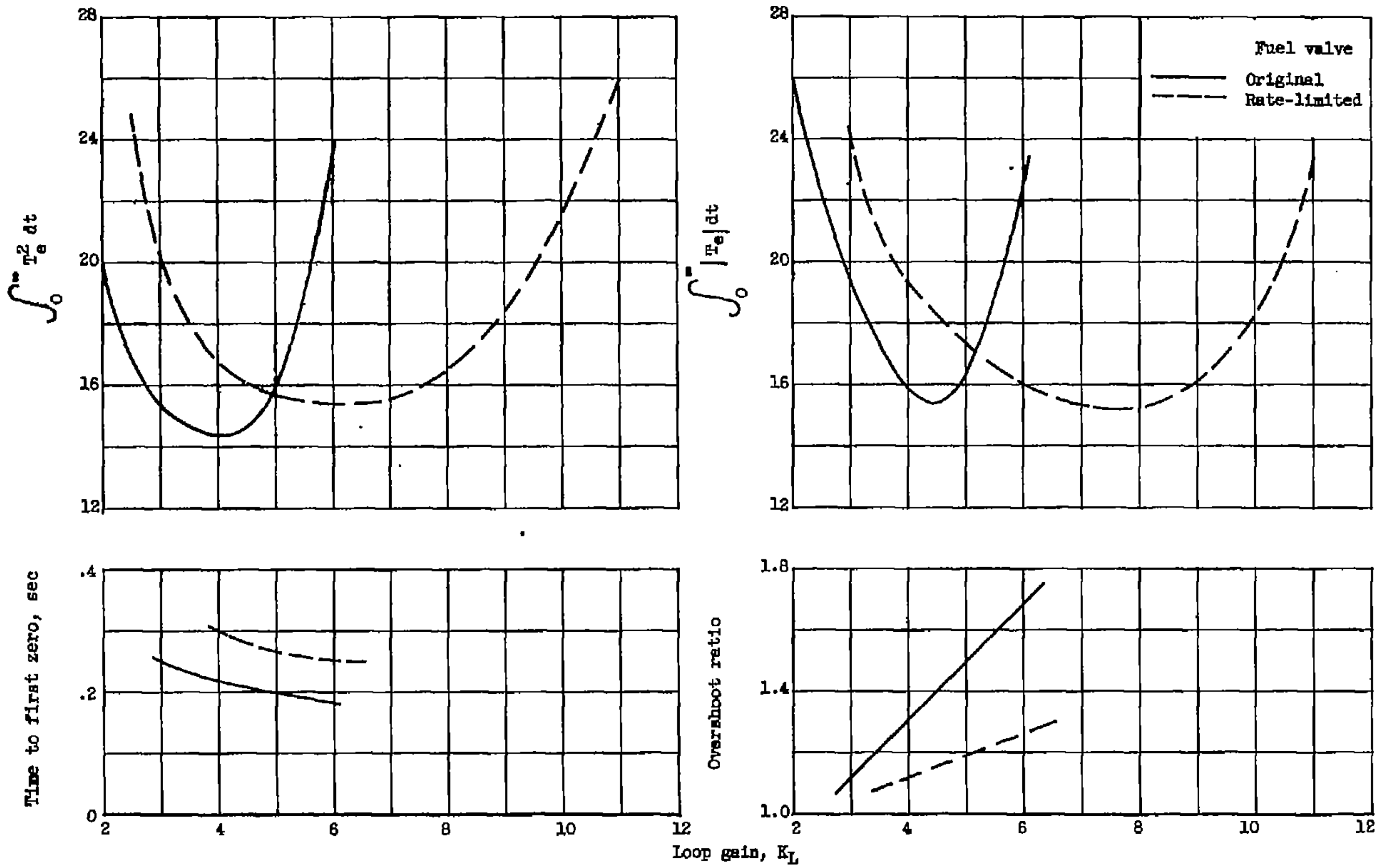
Figure 13. - Continued. Comparison of transient-temperature-response criteria for rate-limited and original fuel-flow control systems.

4212



(c) Control integral time constant, 1.0 second.

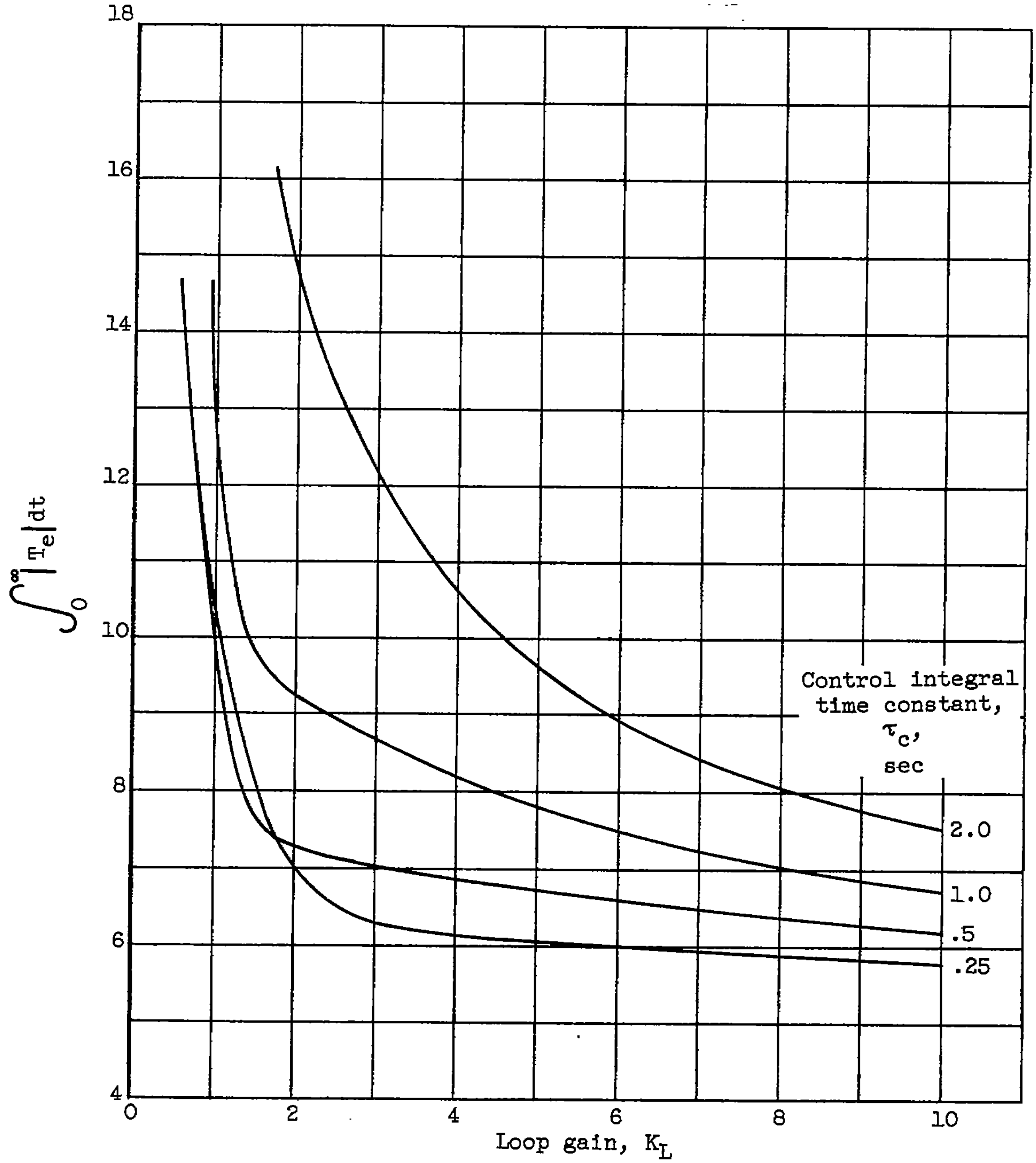
Figure 13. - Continued. Comparison of transient-temperature-response criteria for rate-limited and original fuel-flow control systems.



(d) Control integral time constant, 2.0 seconds.

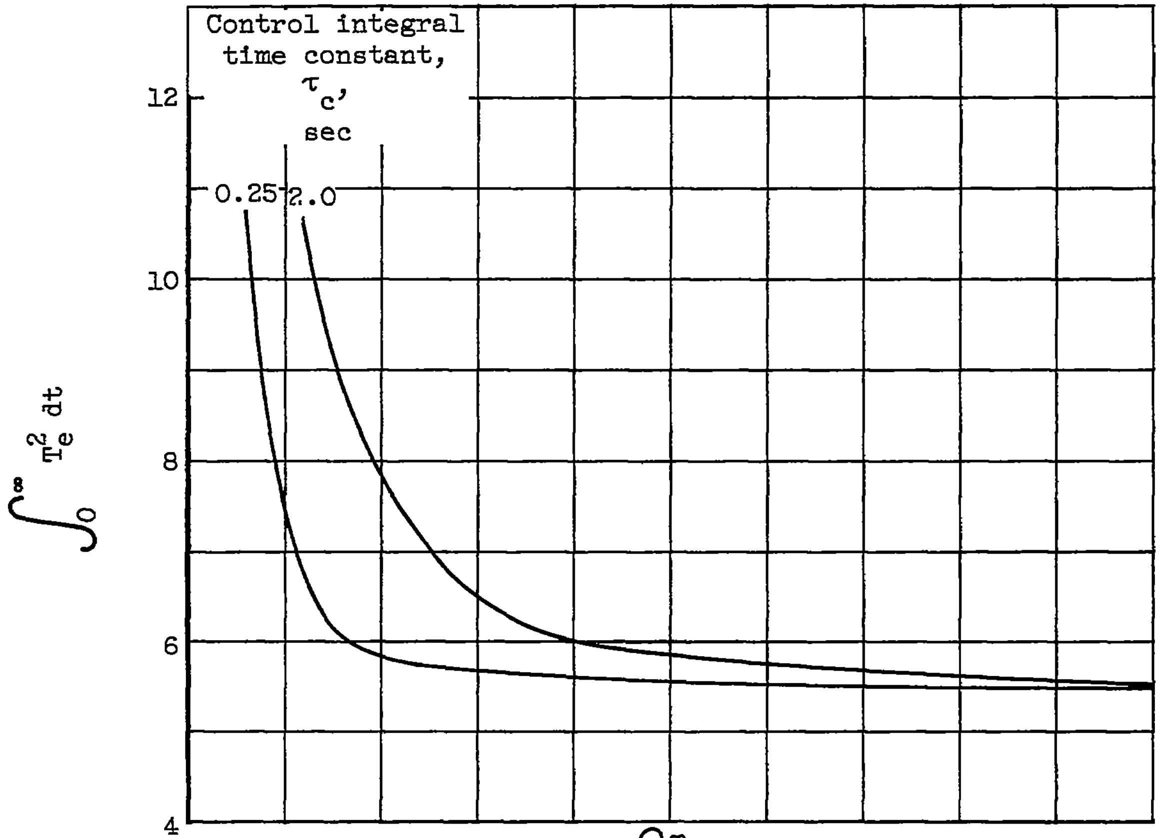
Figure 13. - Concluded. Comparison of transient-temperature-response criteria for rate-limited and original fuel-flow control systems.

4212

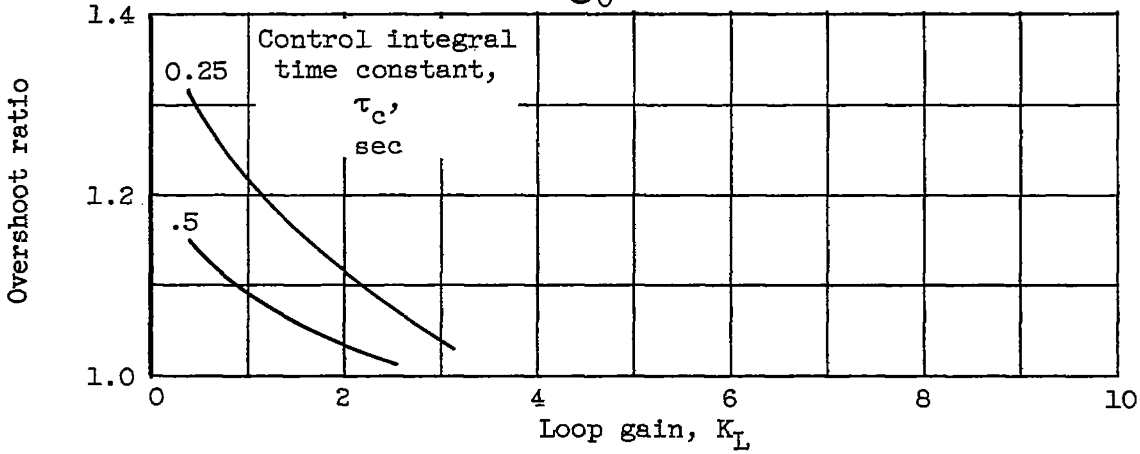


(a) $\int_0^\infty |T_e| dt$.

Figure 14. - Criteria for evaluating transient temperature response for temperature-area control system. Lower-area excursion limit, 85 percent of rated area.



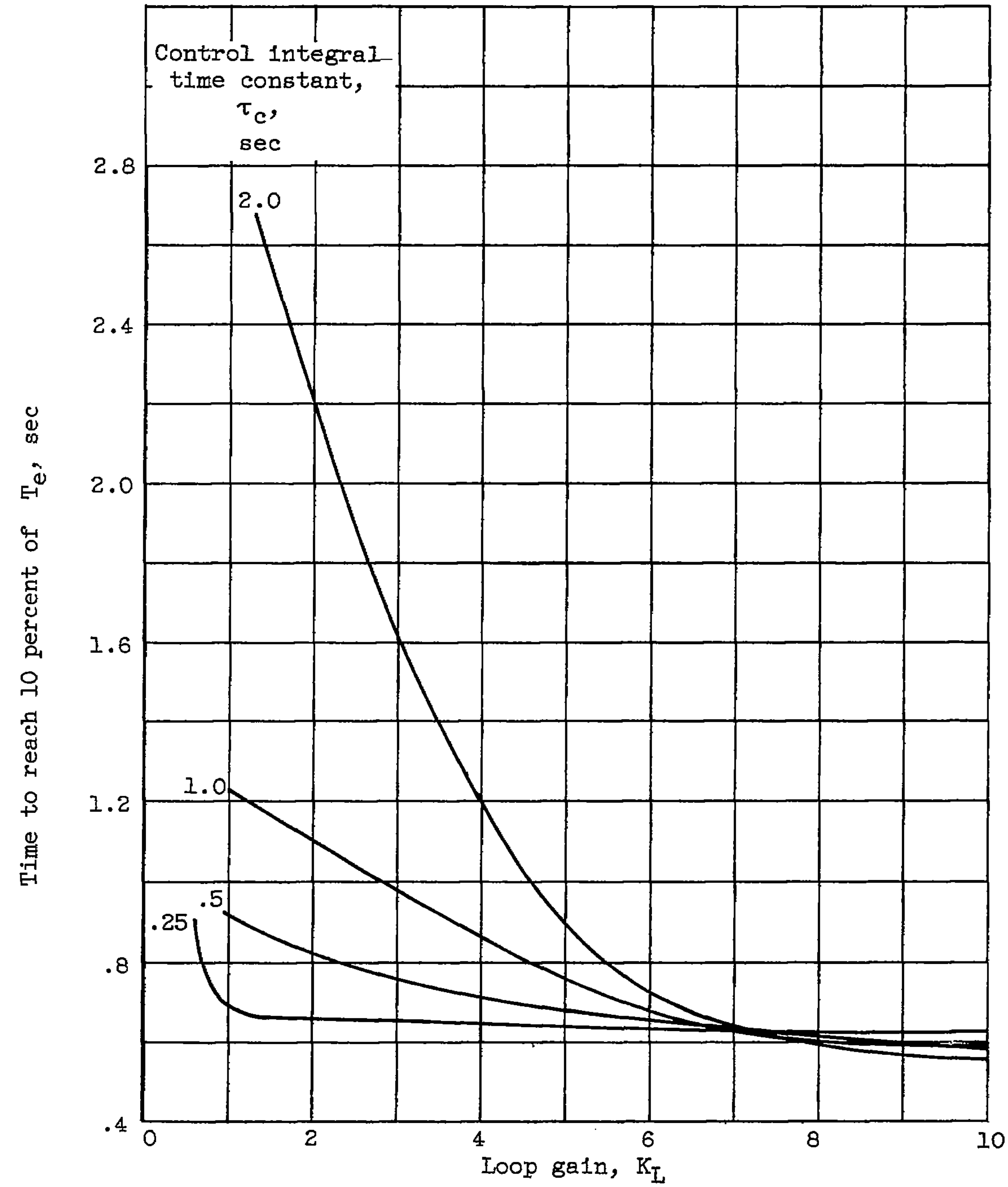
(b) $\int_0^\infty T_e^2\,dt$.



(c) Overshoot ratio.

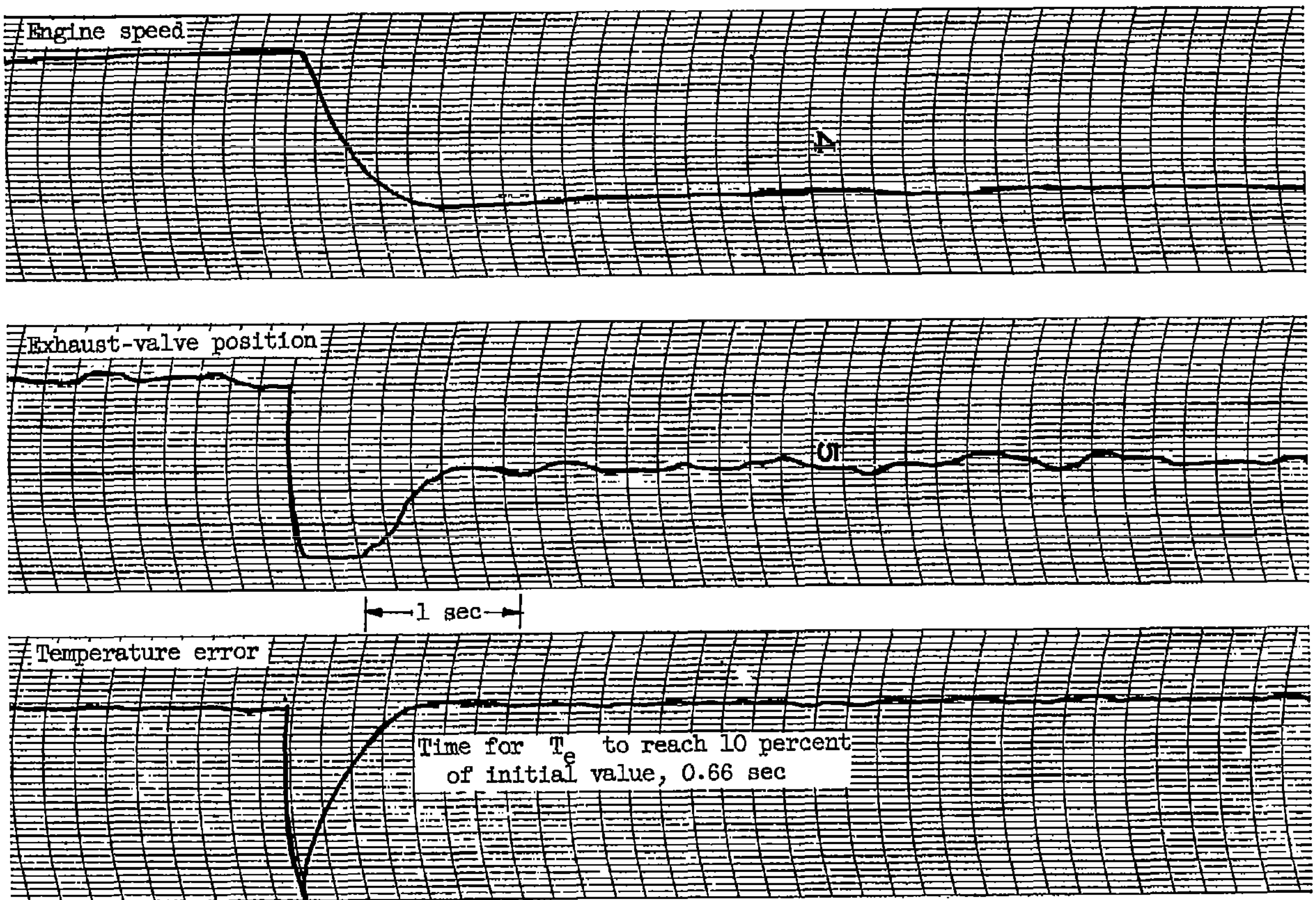
Figure 14. - Continued. Criteria for evaluating transient temperature response for temperature-area control system. Lower-area excursion limit, 85 percent of rated area.

4212



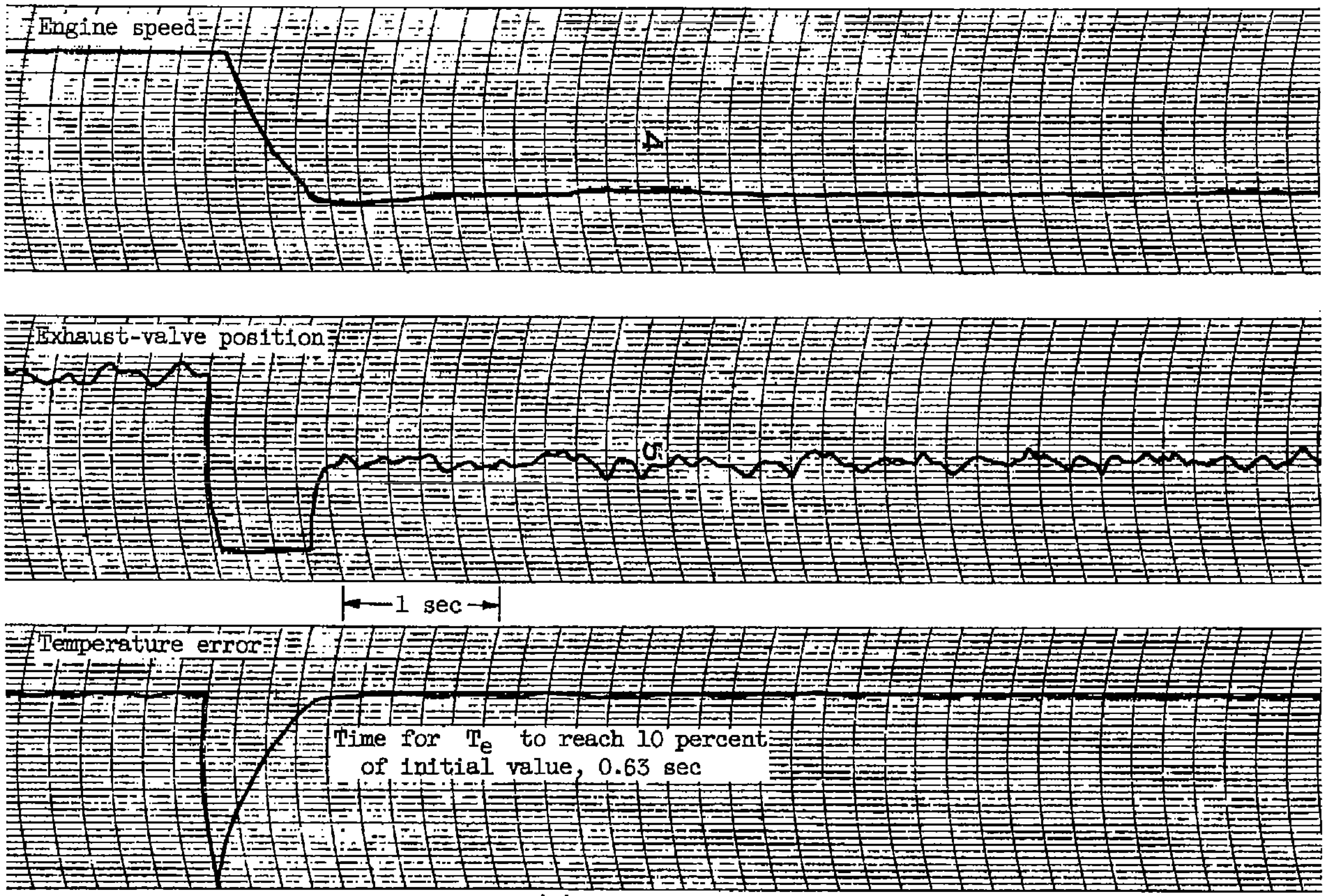
(d) Time to reach 10 percent of initial temperature error.

Figure 14. - Concluded. Criteria for evaluating transient temperature response for temperature-area control system. Lower-area excursion limit, 85 percent of rated area.



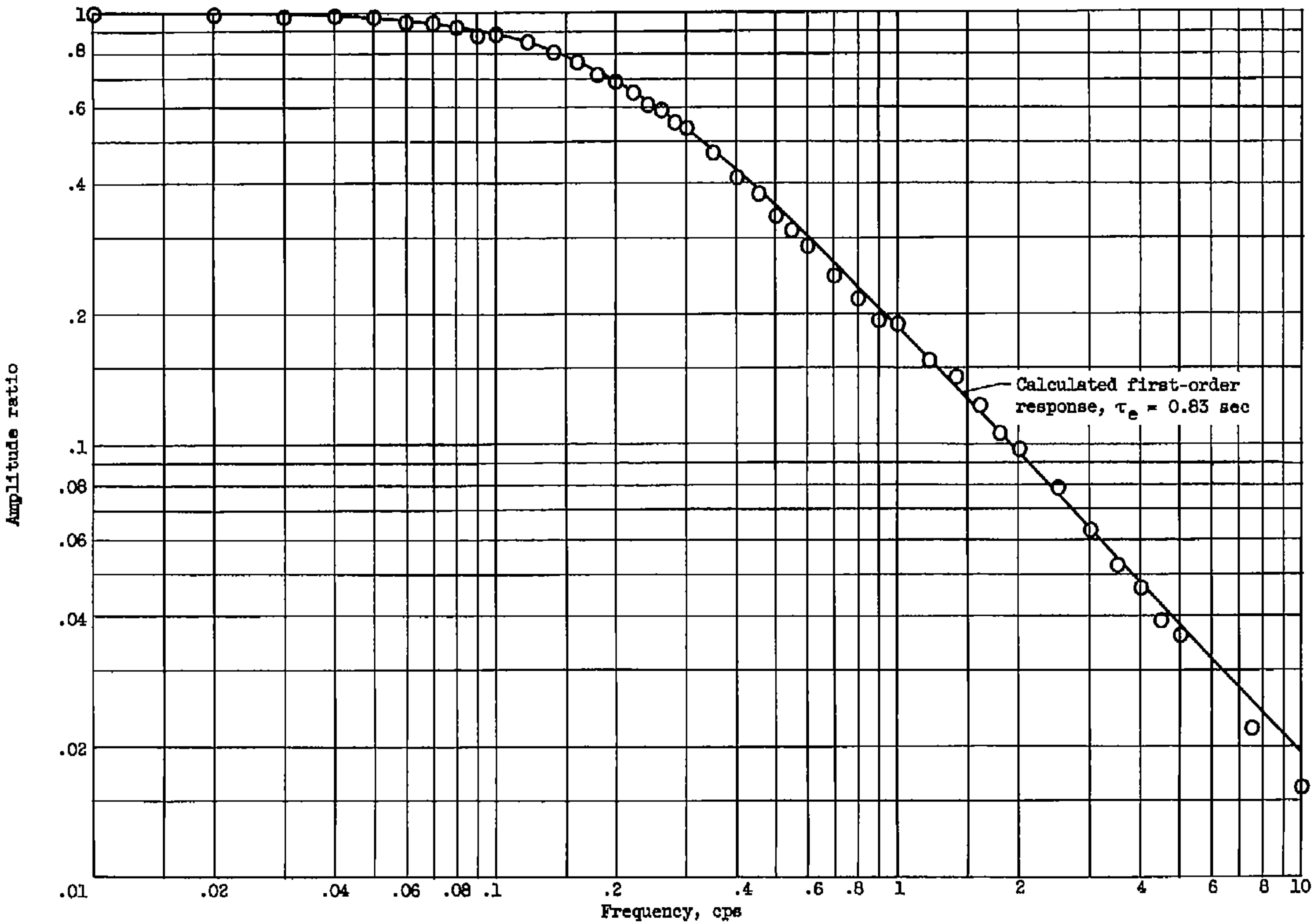
(a) Loop gain, 3.74.

Figure 15. - Transient responses to step increase in reference temperature for temperature-area control system. Control integral time constant, 0.25 second.



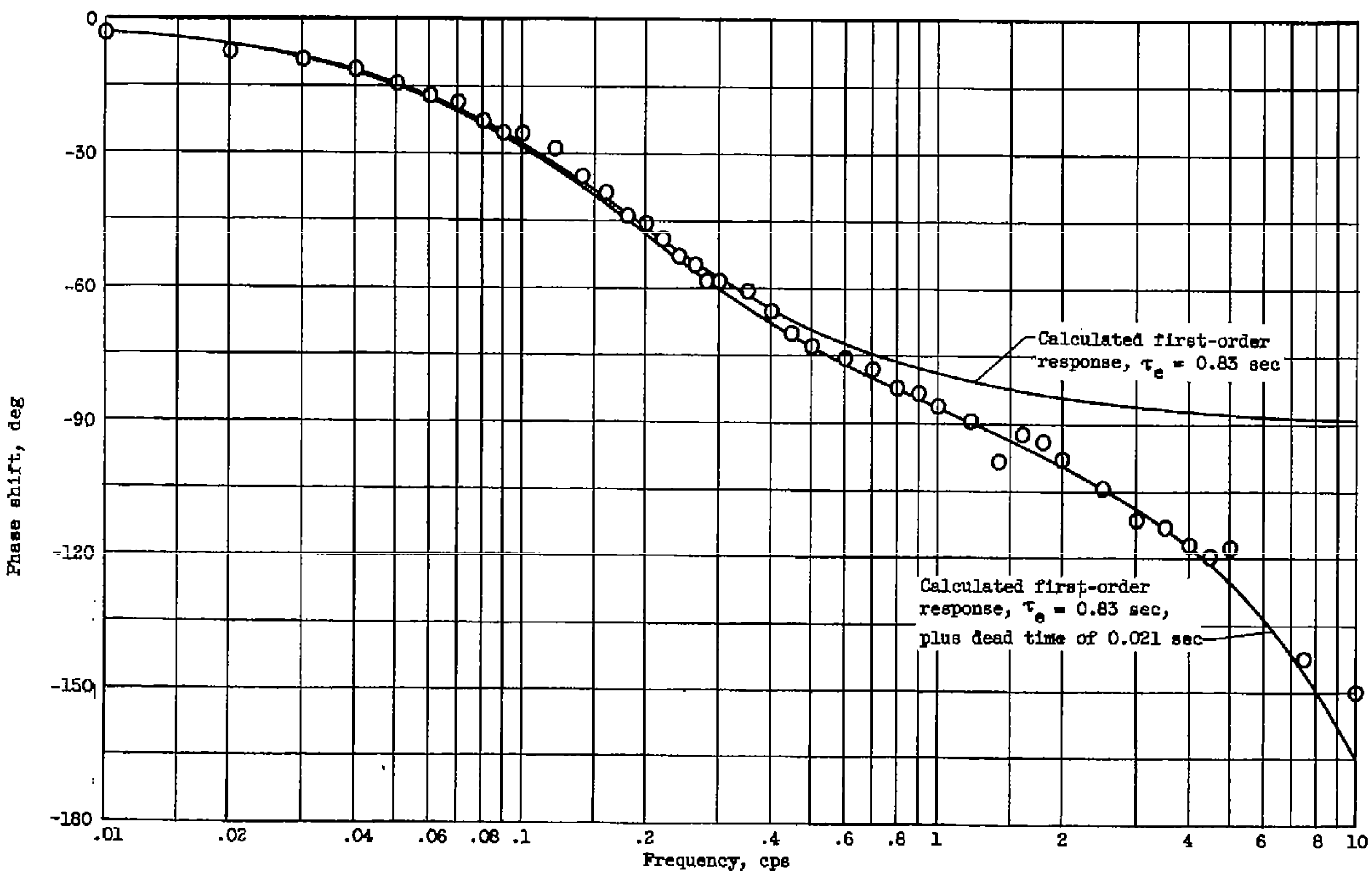
(b) Loop gain, 9.96.

Figure 15. - Concluded. Transient responses to step increase in reference temperature for temperature-area control system. Control integral time constant, 0.25 second.



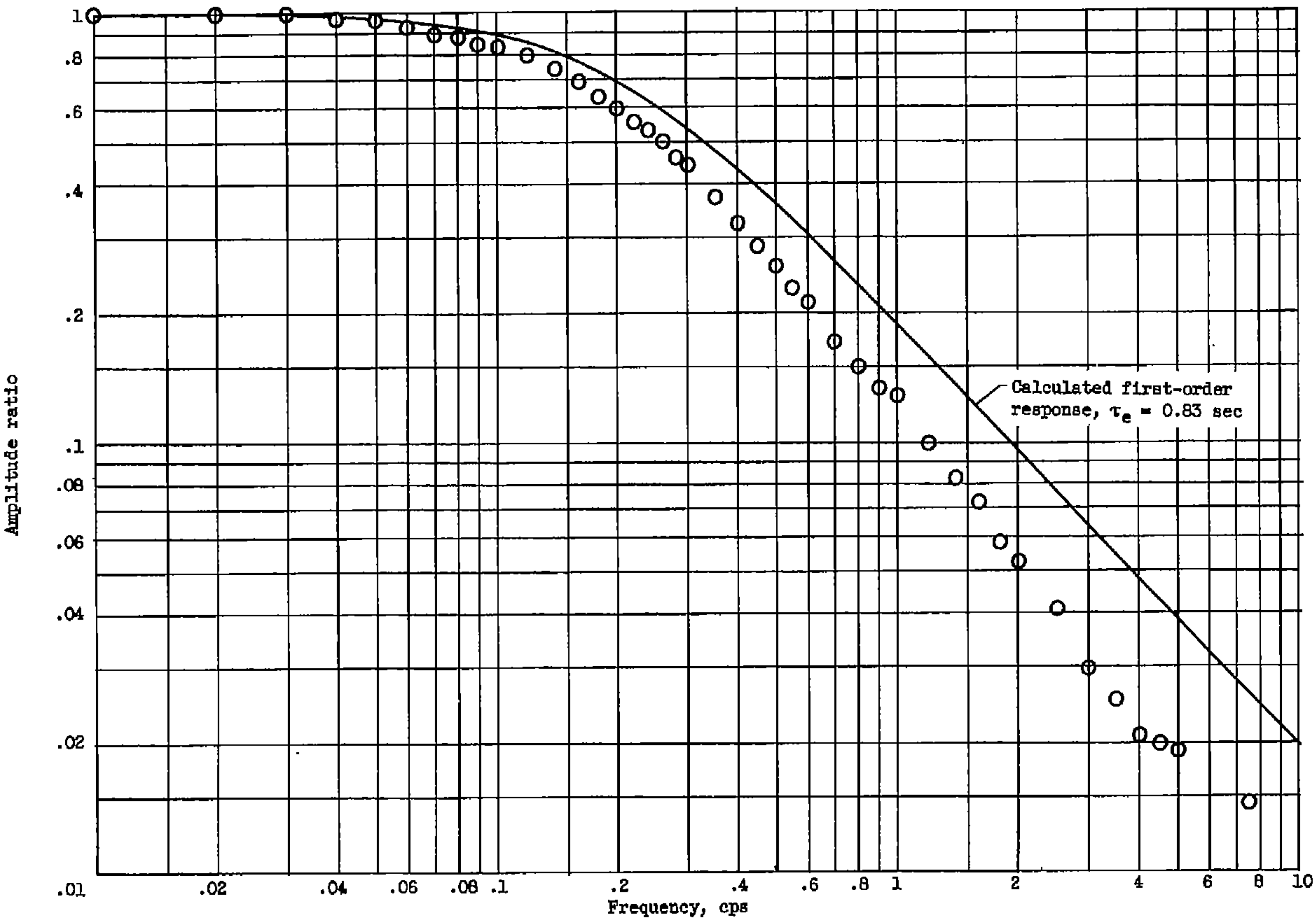
(a) Amplitude ratio.

Figure 16. - Frequency response of engine speed to exhaust-nozzle area at constant fuel flow. Speed variation, ±3 percent of rated speed; operating point, 95.5 percent of rated speed.



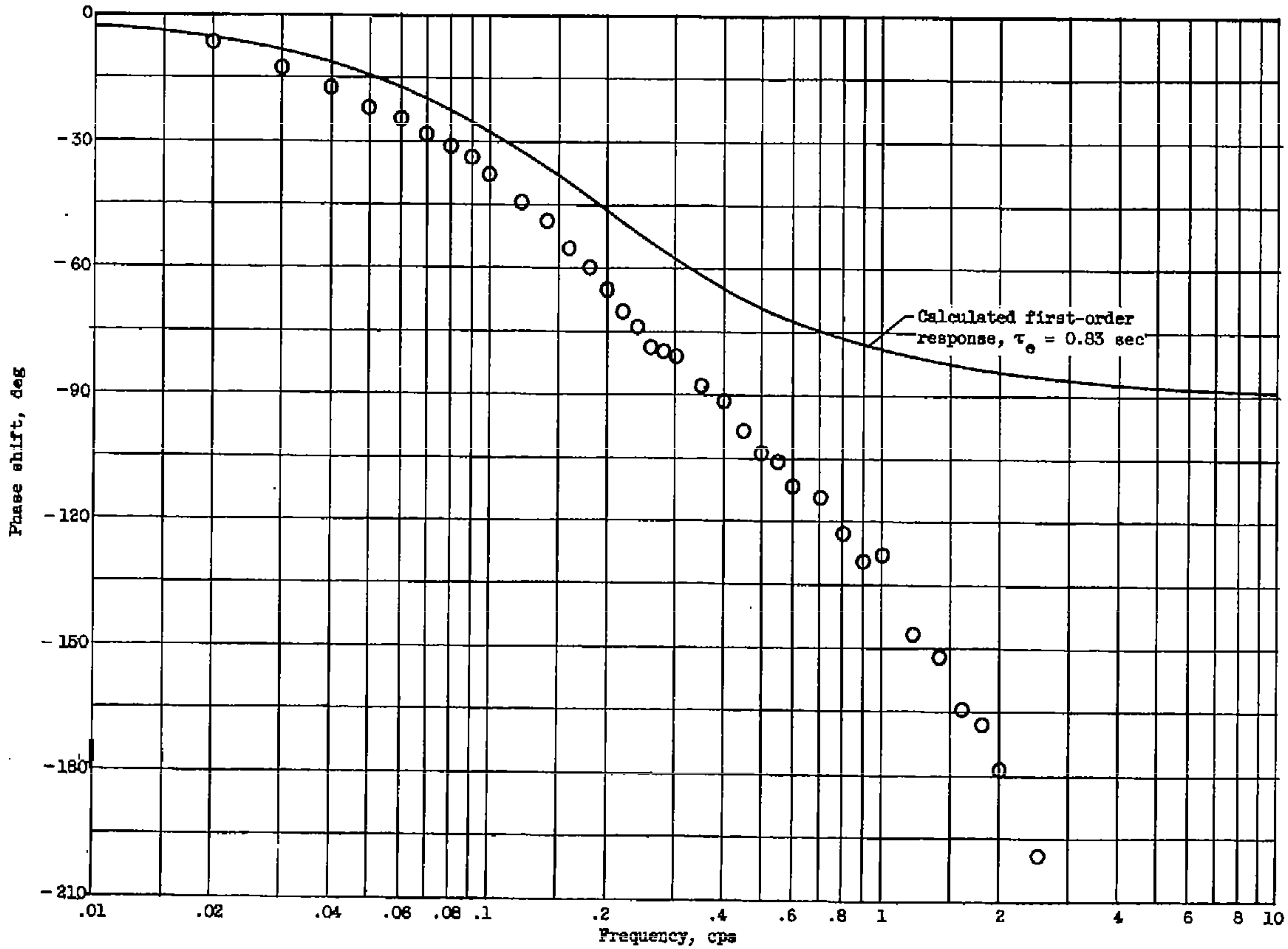
(b) Phase shift.

Figure 16. - Concluded. Frequency response of engine speed to exhaust-nozzle area at constant fuel flow. Speed variation, ±3 percent of rated speed; operating point, 95.5 percent of rated speed.



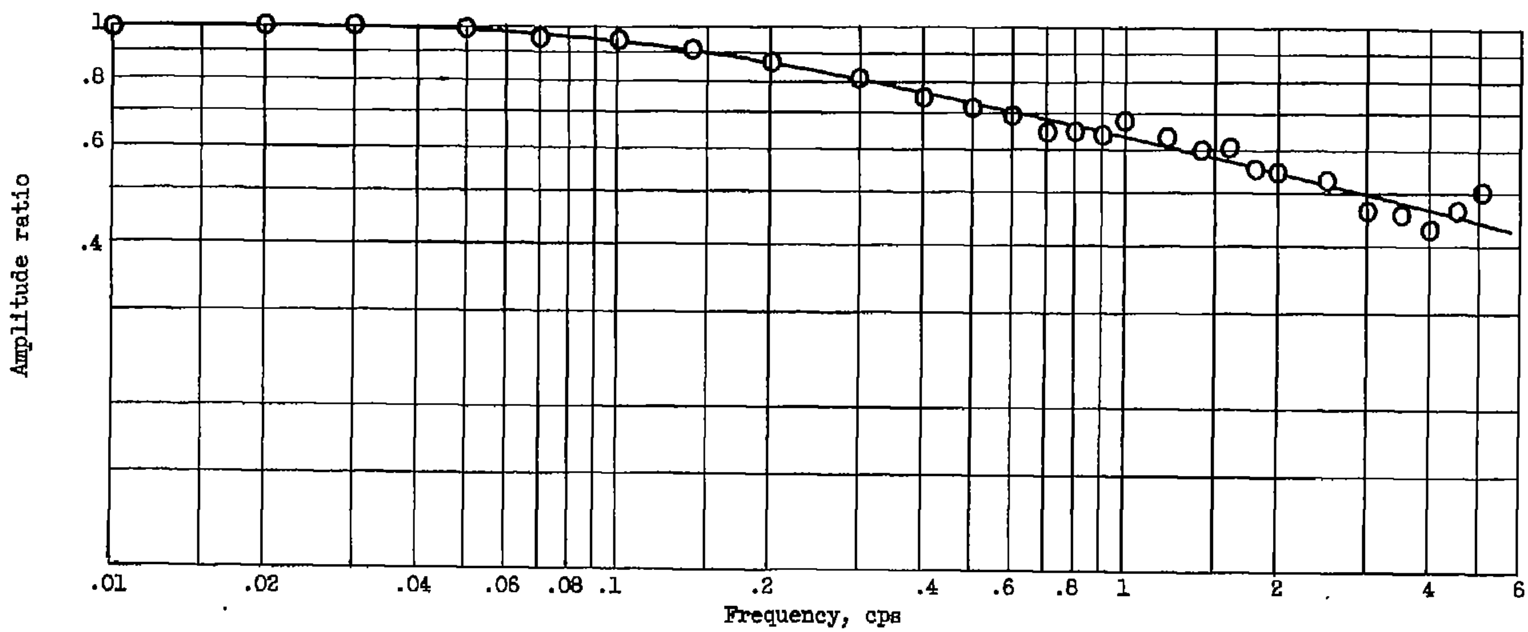
(a) Amplitude ratio.

Figure 17. - Frequency response of engine speed to indicated fuel flow at constant area. Speed variation, ±3 percent of rated speed; operating point, 95.5 percent of rated speed.



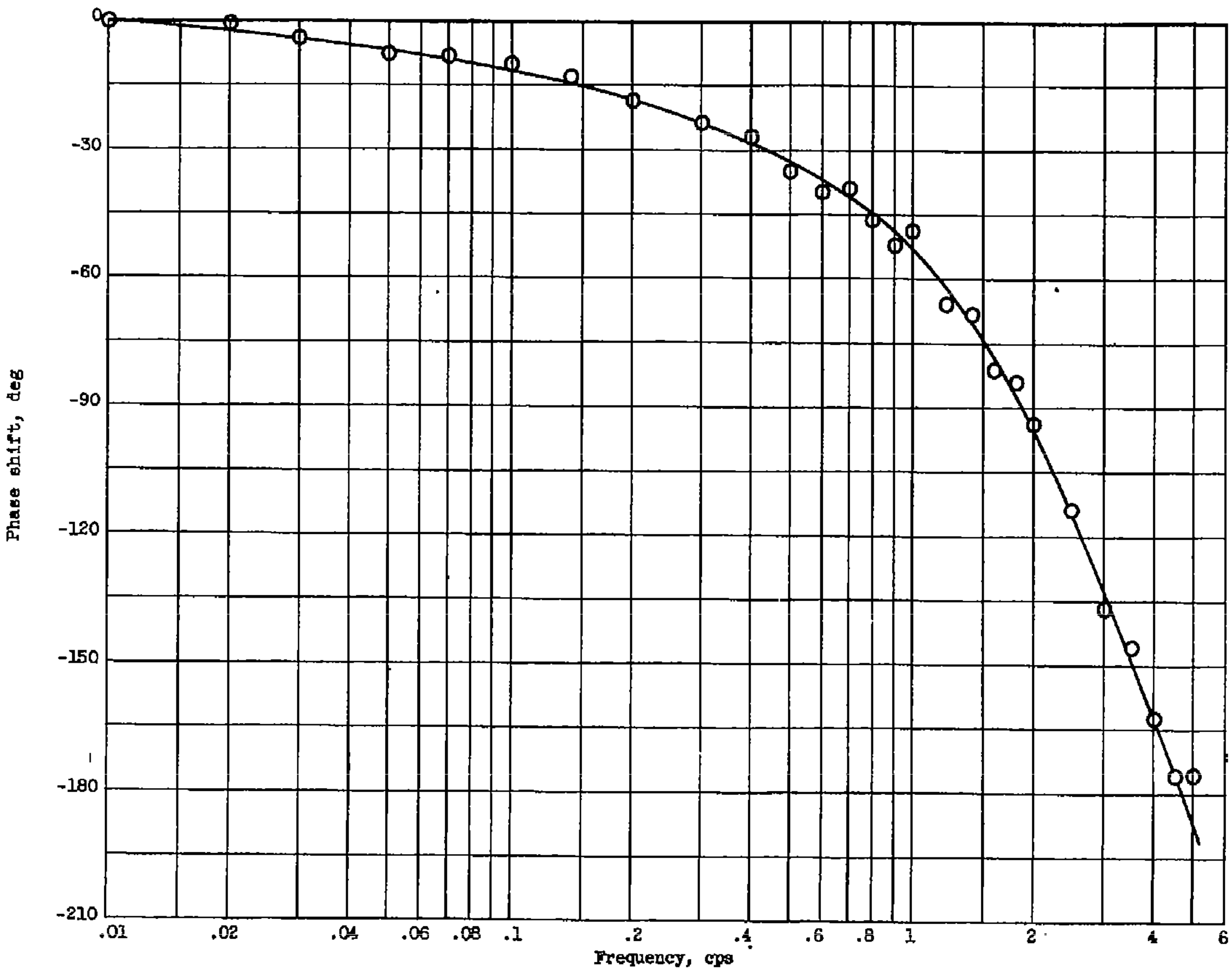
(b) Phase shift.

Figure 17. - Concluded. Frequency response of engine speed to indicated fuel flow at constant area. Speed variation, ±3 percent of rated speed; operating point, 95.5 percent of rated speed.



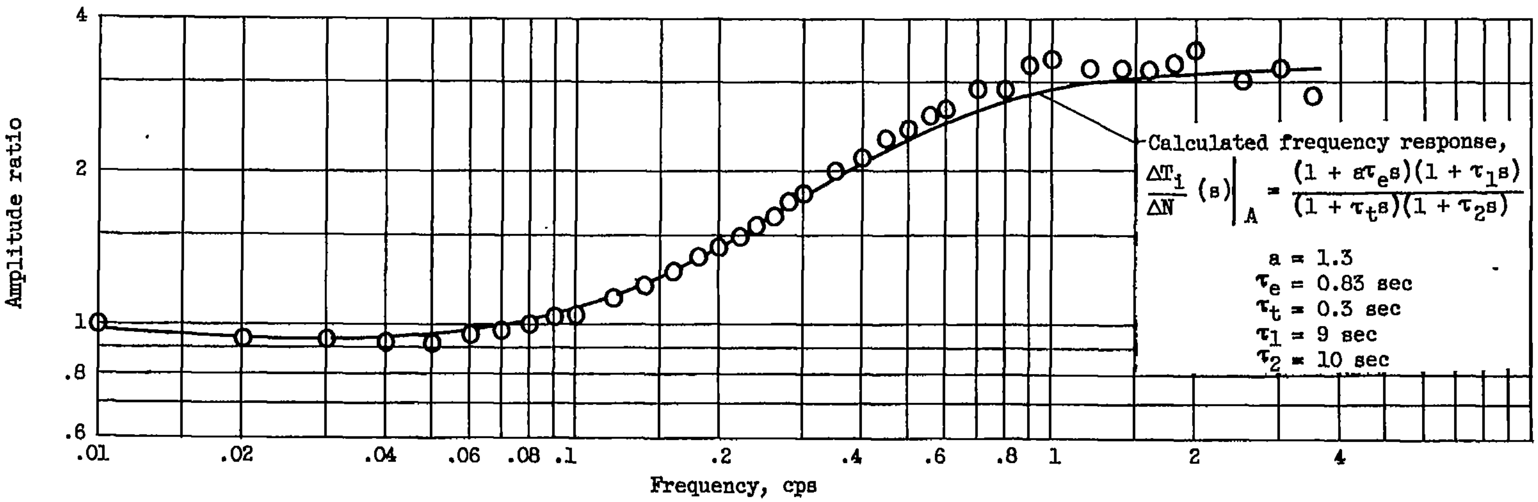
(a) Amplitude ratio.

Figure 18. - Frequency response of fuel-distribution-system and combustion dynamics $G(s)e^{-t_{d,f}s}$.



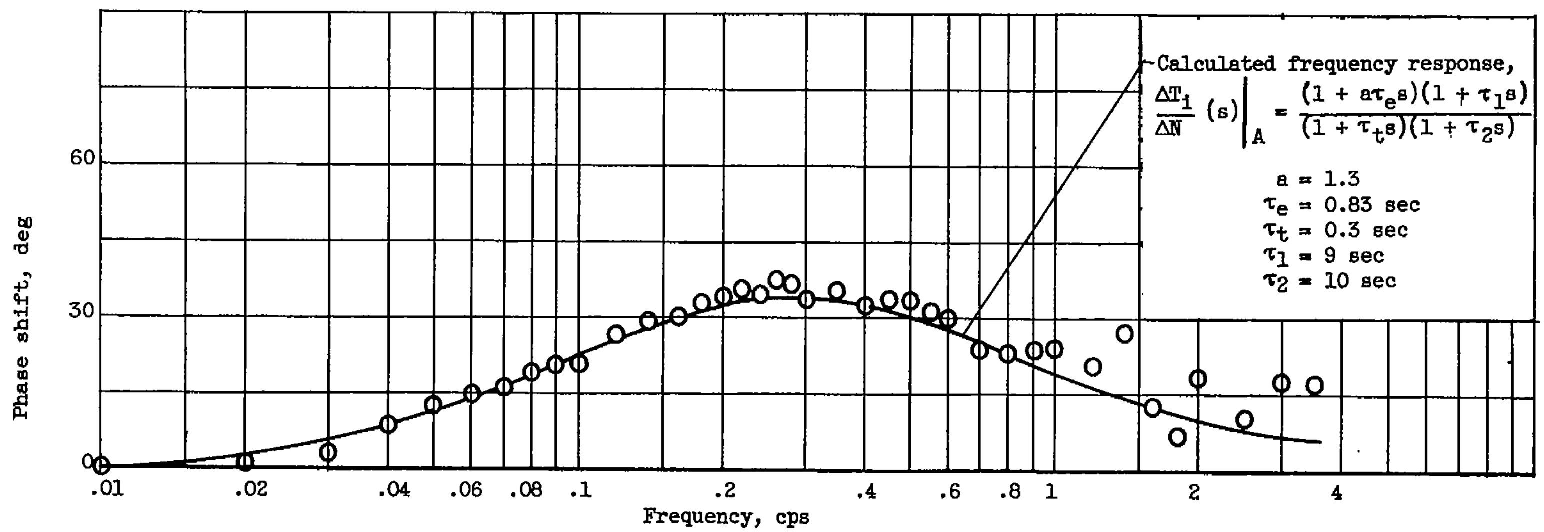
(b) Phase shift.

Figure 18. - Concluded. Frequency response of fuel-distribution-system and combustion dynamics $G(s)e^{-t_{d,f}s}$



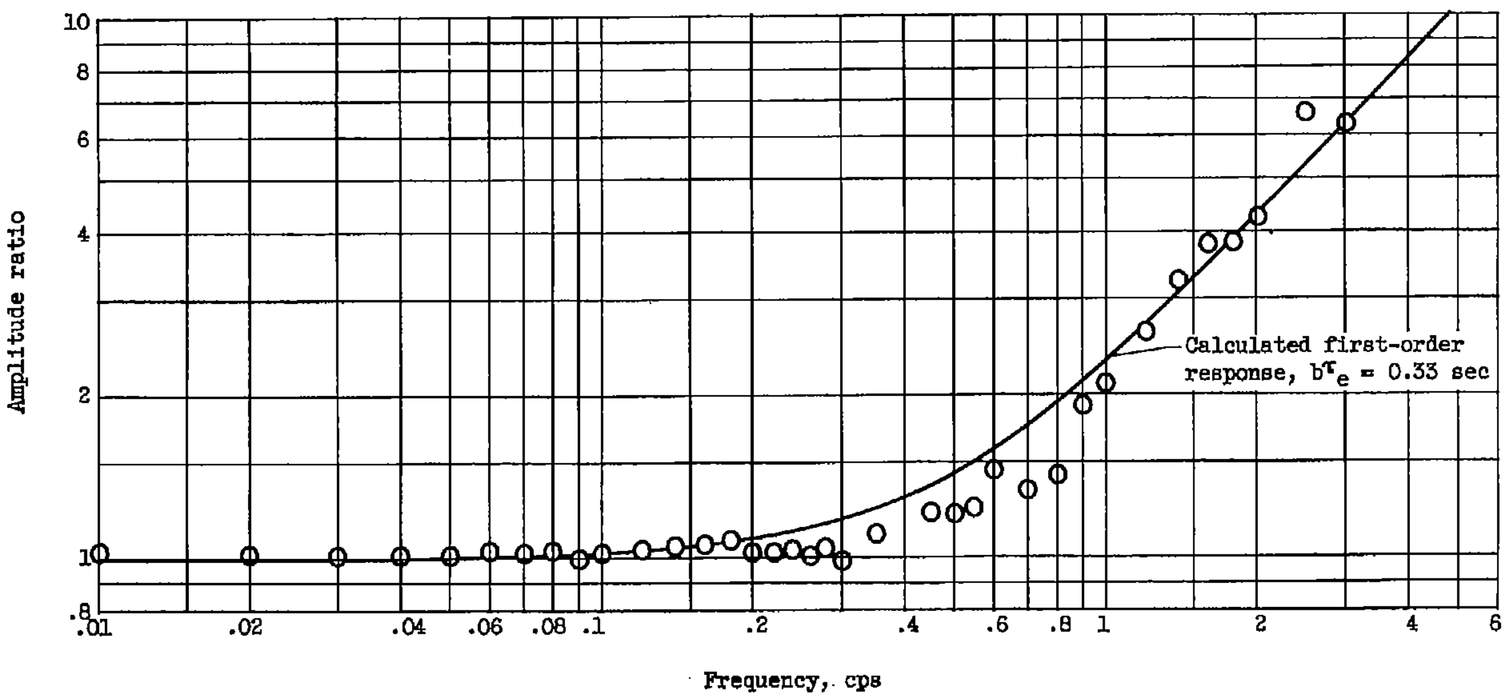
(a) Amplitude ratio.

Figure 19. - Frequency response of indicated tailpipe gas temperature to engine speed at constant area.



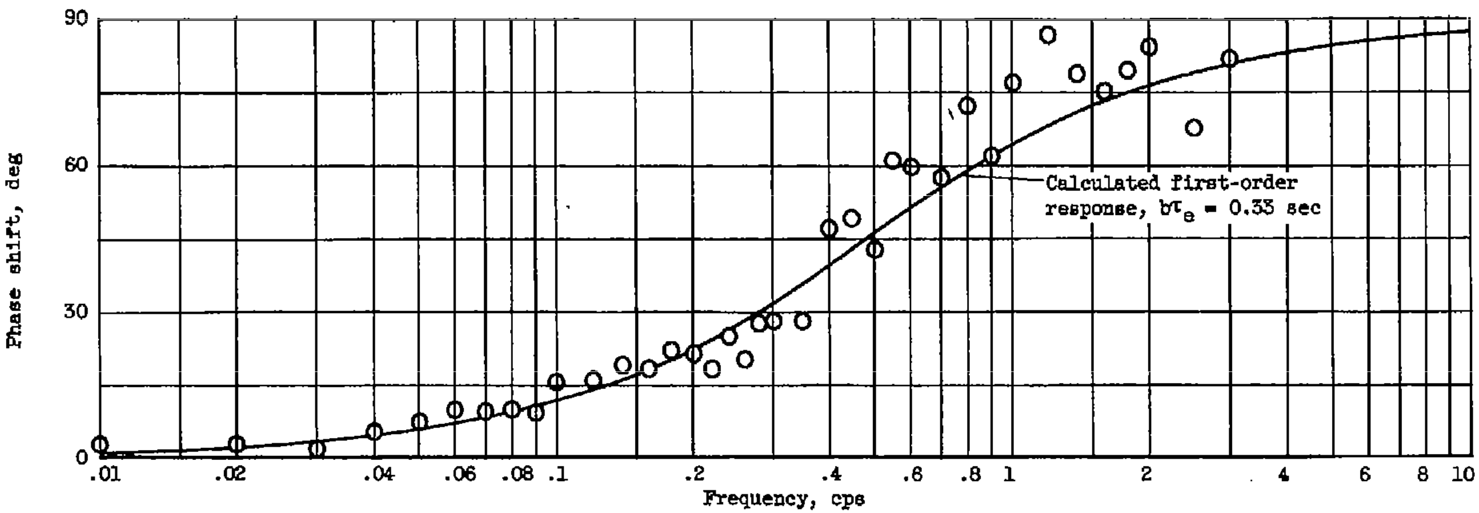
(b) Phase shift.

Figure 19. - Concluded. Frequency response of indicated tailpipe gas temperature to engine speed at constant area.



(a) Amplitude ratio.

Figure 20. - Calculated frequency response of engine lead term $1 + b\tau_e s$.



(b) Phase shift.

Figure 20. - Concluded. Calculated frequency response of engine lead term $1 + b\tau_e s$.

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